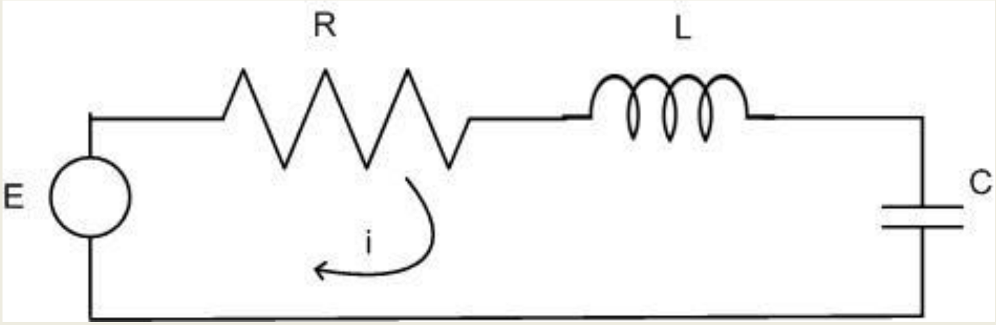
OUTLINE

- Introduction to Analogous System.
- Force-Voltage Analogy.
- Force-Current Analogy.
- Example on Analogous System.
- Mechanical Equivalent Network.

ANALOGOUS SYSTEM

Apply KVL in Series RLC Circuit



$$E = Ri + L\frac{di}{dt} + \frac{1}{C}\int i dt ------(1)$$

or

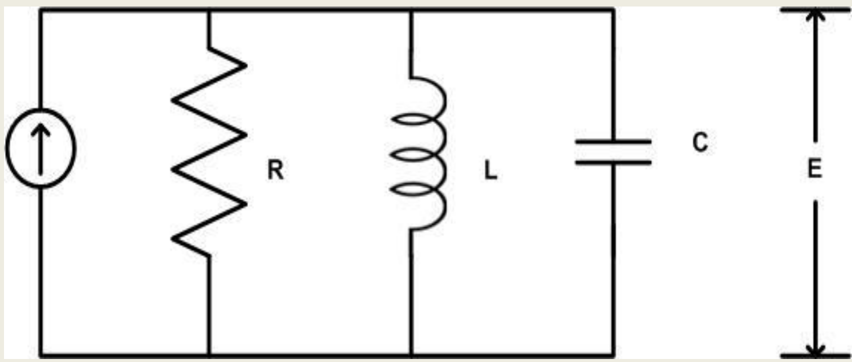
$$E = R\frac{dq}{dt} + L\frac{d^2q}{dt^2} + \frac{1}{C}q -----(2)$$

Consider parallel RLC circuit and apply KCL

$$I = \frac{E}{R} + \frac{1}{L}\int E dt + C\frac{dE}{dt} ------(3)$$

$$\phi = \int E dt, E = \frac{d\phi}{dt}$$

$$I = \frac{1}{R}\left(\frac{d\phi}{dt}\right) + \frac{1}{L}\phi + C\frac{d^2\phi}{dt^2} -----(4)$$



We know the equation of mechanical system

$$F(t) = M\frac{d^2x(t)}{dt^2} + B\frac{dx(t)}{dt} + Kx(t) ------(5)$$

Compare equation(5) with equation(2)

FORCE –VOLTAGE ANALOGY (f-v)

S.NO.	TRANSLATIONAL SYSTEM	ELECTRICAL SYSTEM
1.	Force (F)	Voltage (E)
2.	Mass (M)	Inductance (L)
3.	Stiffness (K), Elastance (1/K)	Reciprocal of C, Capacitance (C)
4.	Damping coefficient (B)	Resistance (R)
5.	Displacement (x)	Charge (q)

Compare equation (5) with equation (4)

FORCE-CURRENT ANALOGY

S.NO.	TRANSLATIONAL SYSTEM	ELECTRICAL SYSTEM
1.	Force (F)	Current (I)
2.	Mass (M)	Capacitance (C)
3.	Damping coefficient (B)	Reciprocal of resistance (1/R) i.e conductance (G)
4.	Stiffness (K), Elastance (1/K)	Reciprocal of inductance (1/L)
5.	Displacement (x) $\dot{x}$	Flux linkage (ϕ)
6.	Velocity	Voltage (E)

For rotational system

$$T(t) = J\frac{d\omega(t)}{dt} + B\frac{d\theta(t)}{dt} + K\theta(t) ------(6)$$

Torque-voltage (T-V) analogy

Compare equation(2) with equation (6)

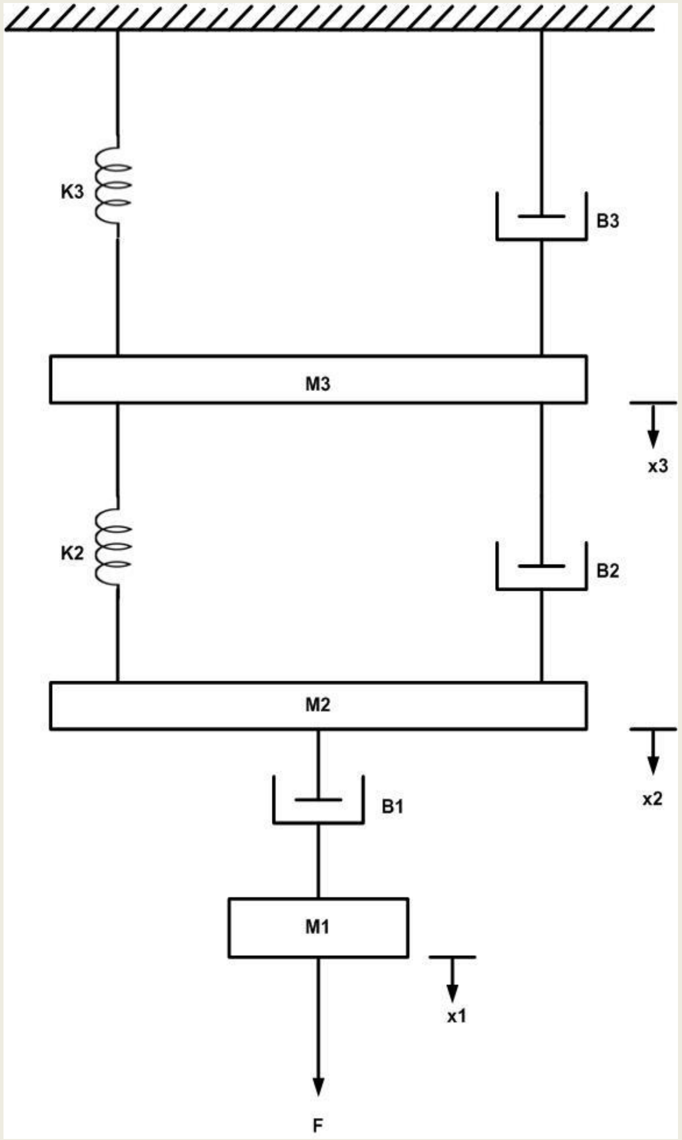
S.NO.	ROTATIONAL SYSTEM	ELECTRICAL SYSTEM
1.	Torque (T)	Voltage (E)
2.	Moment of inertia (J)	Inductance (L)
3.	Damping coefficient (B)	Resistance (R)
4.	Stiffness (K), Elastance (1/K)	Reciprocal of capacitance (1/C). Capacitance (C)
5.	Angular displacement (θ)	Charge (q)
6.	Angular velocity (ω)	Current (I)

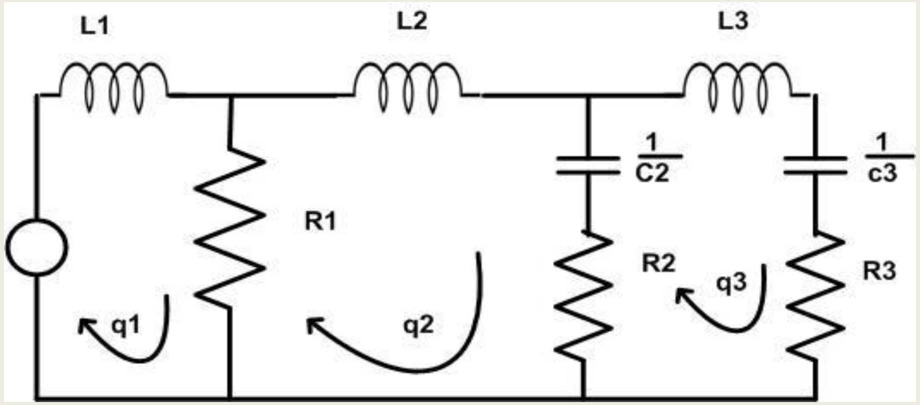
Compare equation (4) with equation (6)

TORQUE(T)-CURRENT (I) ANALOGY

S.NO.	ROTATIONAL SYSTEM	ELECTRICAL SYSTEM
1.	Torque (T)	Current (I)
2.	Moment of inertia (J)	Capacitance (C)
3.	Damping coefficient (B)	Reciprocal of resistance (R), conductance (G)
4.	Stiffness (K), Elastance (1/K)	Reciprocal of inductance (1/L)
5.	Angular displacement (ω)	Flux linkage (ϕ)

Draw the analogous electrical network of the given fig. using f-v analogy





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MECHANICAL EQUIVALENT NETWORK

PROCEDURE TO DRAW THE MECHANICAL EQUVALENT NETWORK:

Step 1: Draw a reference line.

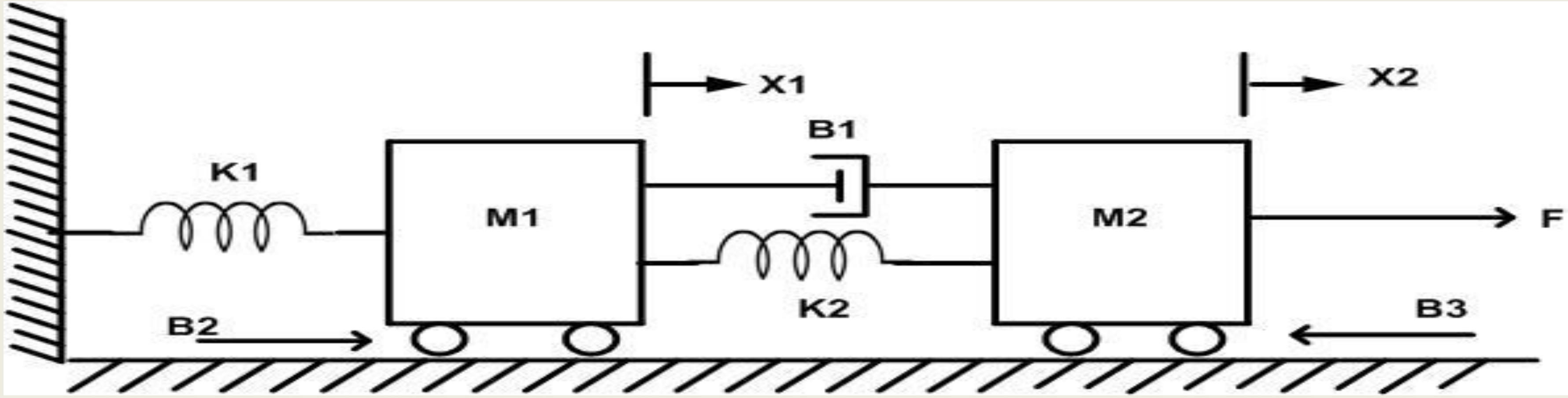
Step 2: Corresponding to the displacement x_1 x_2 ….select the nodes.

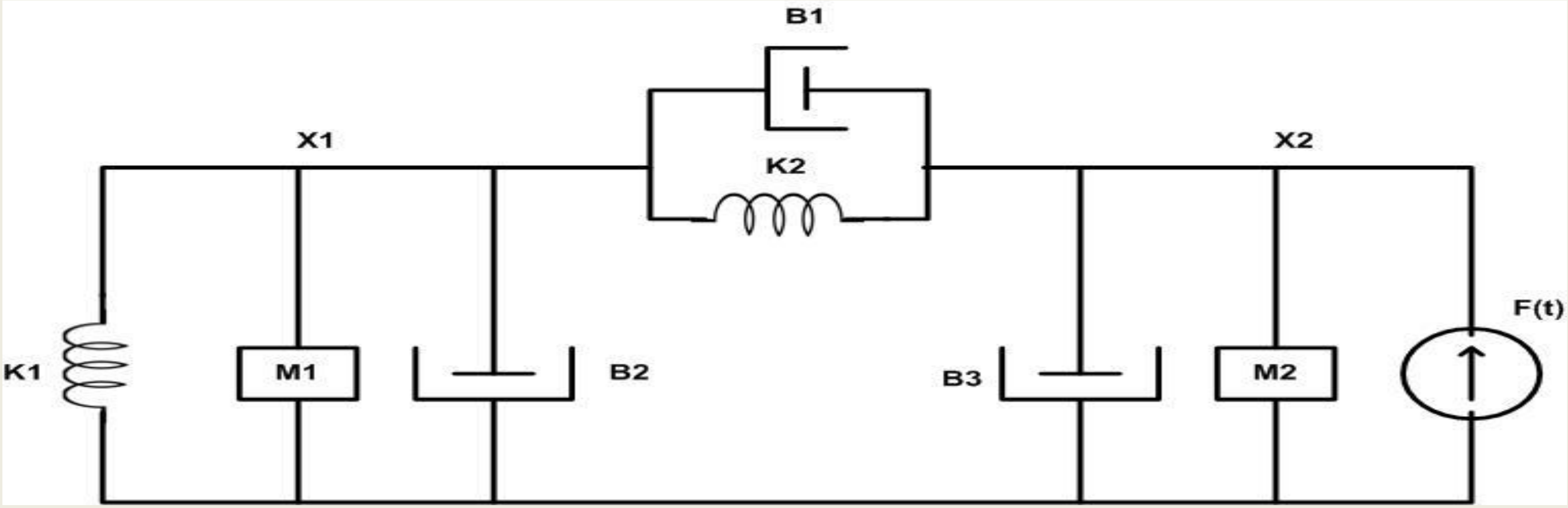
Step 3: Connect one end of masses to the reference line

Step 4: Connect other elements of the system to the nodes.

Step 5: Apply nodal analysis, write the system equations.

Example: Draw the mechanical network of the given system





THANK YOU FOR YOUR ATTENTION

OUTLINE

- Advantages and disadvantages of block diagram.
- Terminology.
- Closed loop.
- How to draw the block diagram.
- Block reduction Technique.

BLOCK DIAGRAM REPRESENTATION

BLOCK DIAGRAM:

a. Block diagram is a pictorial representation of the system.

b. Block diagram represent the relationship between input and output.

c. Any complicated system can be represented by connecting the different block.

d. Each block is completely characterized by a transfer function.

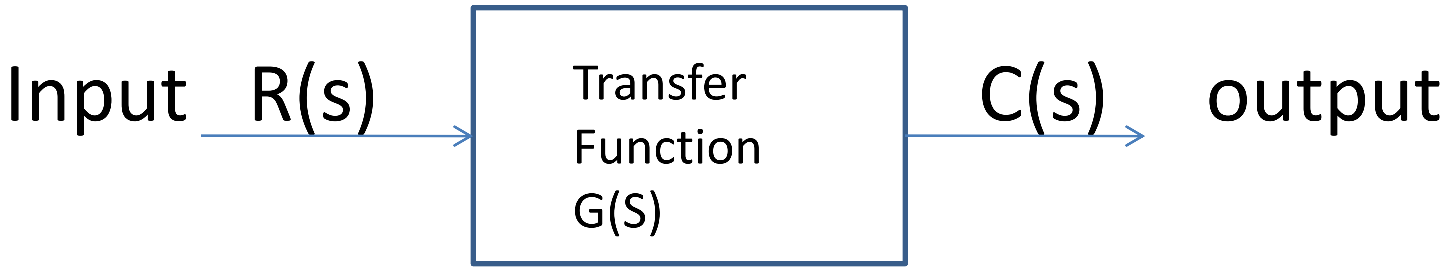
ADVANTAGES OF BLOCK DIAGRAM:

a. A block may represent a single component or a group of components.
b. Performance of a system can be determined by block diagram.
c. Block diagram is helpful for designing and analysis of a system.
d. Construction of block diagram is simple for any complicated system.
e. The operation of any system can be easily observed by block diagram representation.

DISADVANTAGES OF BLOCK DIAGRAM:

a. Source of energy is not shown in the diagram.
b. Block diagram does not give any information about the physical construction of the system.
c. Applicable only for linear time invariant system.
d. It requires more time.
e. It is not a systematic method.
f. For a given system, the block diagram is not unique.

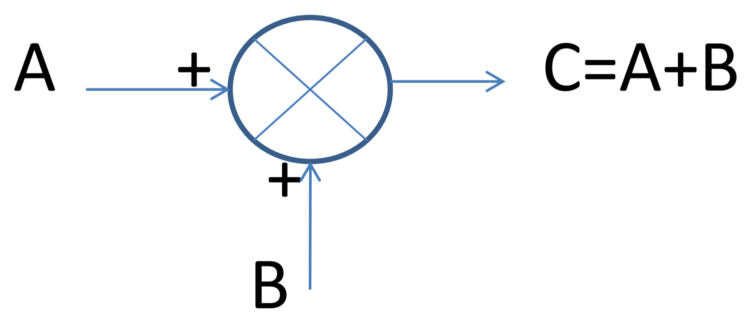
Following figure shows the different components of block diagram



The arrow pointing towards block indicates input R(s) and arrow head leading away from the block represent the output C(s).

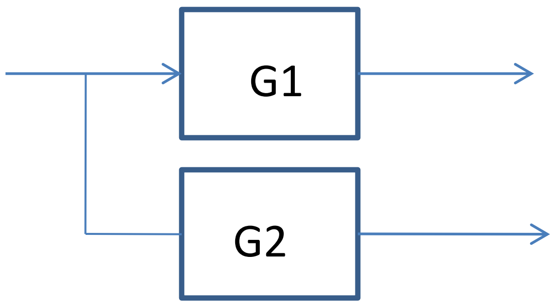
$$C(s) = G(s)R(s) \quad \text{or} \quad G(s) = \frac{C(s)}{R(s)}$$

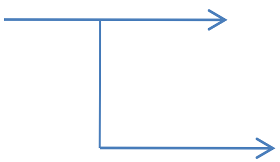
The flow of system variables from one block to another block is represented by the arrow. In addition to this, the sum of the signals or the difference of the signals are represented by a summing point as shown in figure.



The plus or minus sign at each arrow head indicates whether the signal is to be added or subtracted.

Application of one input source to two or more block is represented by a take off point. Take off point also known as branch point, shown below





Forward path: The direction of flow of signal from input to output.

Feedback path: The direction of flow of signal is from output to input.

BLOCK DIAGRAM OF A CLOSED LOOP SYSTEM

r(t) = reference input signal

c(t)= controlled output

e(t)= error signal

b(t)= feedback signal

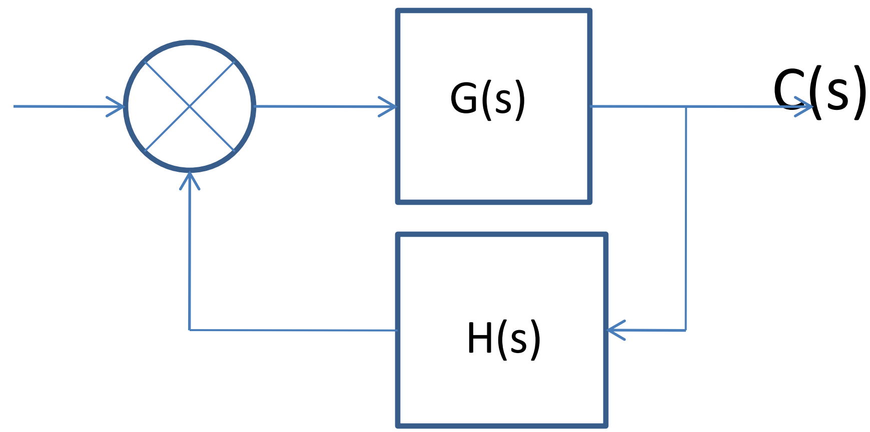
R(s)=Laplace transform of input

C(s)= Laplace transform of output

B(s) Laplace transform of feedback signal

E(s) Laplace transform of error signal

Block diagram



From the figure

$$C(s) = G(s)E(s) \underline{\hspace{5cm}} (1)$$

$$B(s) = C(s)H(s) \underline{\hspace{5cm}} (2)$$

The error signal is $E(s) = R(s) - B(s)$

From above equations we get

$$C(s) = G(s)\left[R(s) - B(s)\right]$$

$$C(s) = G(s)R(s) - G(s)B(s)$$

$$C(s) = G(s)R(s) - G(s)H(s)C(s)$$

$$C(s)\left[1 + G(s)H(s)\right] = G(s)H(s)$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} \underline{\hspace{8cm}} (3)$$

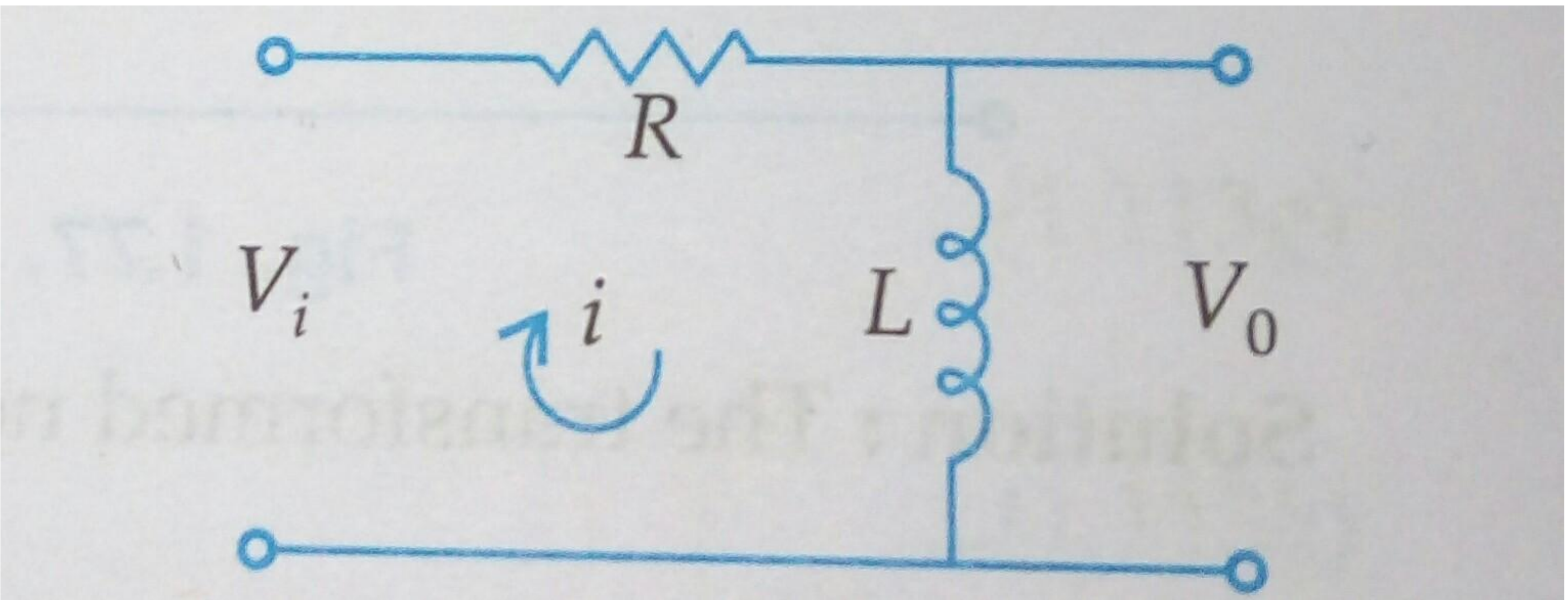
Equation (3) is for negative feedback.

For positive feedback equation (3) becomes

$$\frac{C(s)}{R(s)}=\frac{G(s)}{1-G(s)H(s)}$$

HOW TO DRAW BLOCK DIAGRAM ?

Draw the block diagram of the given circuit.



Apply KVL

$$v_i = Ri + L\frac{di}{dt} - - - - - - - - - (1)$$

$$v_o = L\frac{di}{dt} - - - - - - - - - (2)$$

Laplace Transform of equations (1) & (2) with initial condition zero.

$$V_i(s) = I(s)R + sLI(s) - - - - - - (3)$$

$$V_i(s) = I(s)(R + sL) - - - - - - - - (4)$$

$$V_o(s) = sLI(s) - - - - - - - - - - - (5)$$

From equations (4) & (5)

$$\frac{V_o(s)}{V_i(s)} = \frac{sL}{R + sL}$$

From fig.

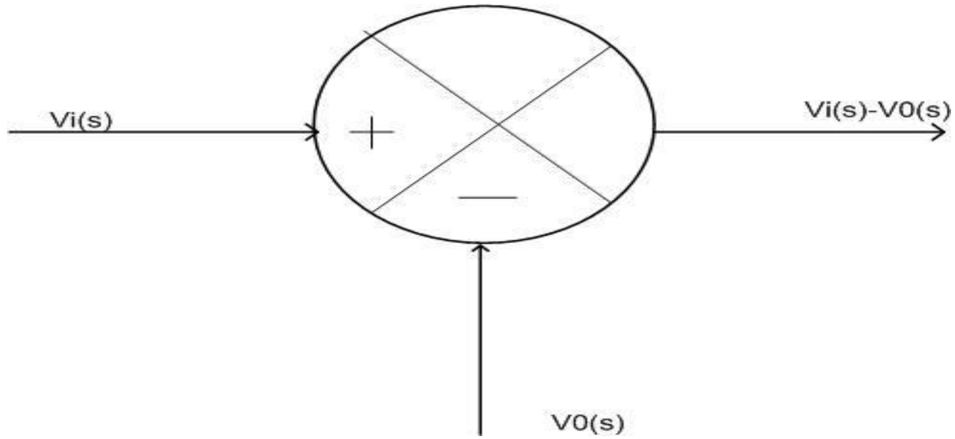
$$i = \frac{v_i - v_o}{R} --- (6)$$

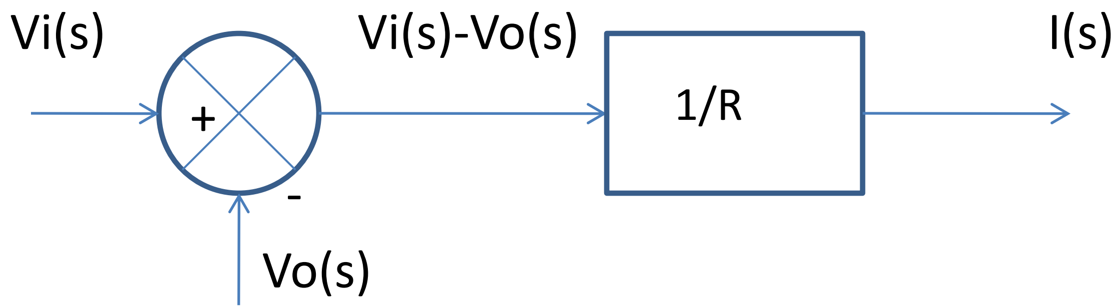
$$v_o = L\frac{di}{dt} --- (7)$$

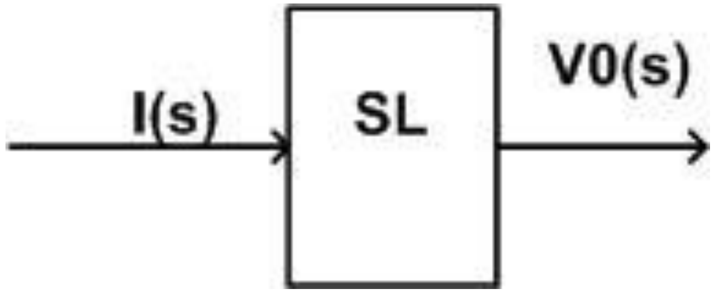
Laplace transform of above equations

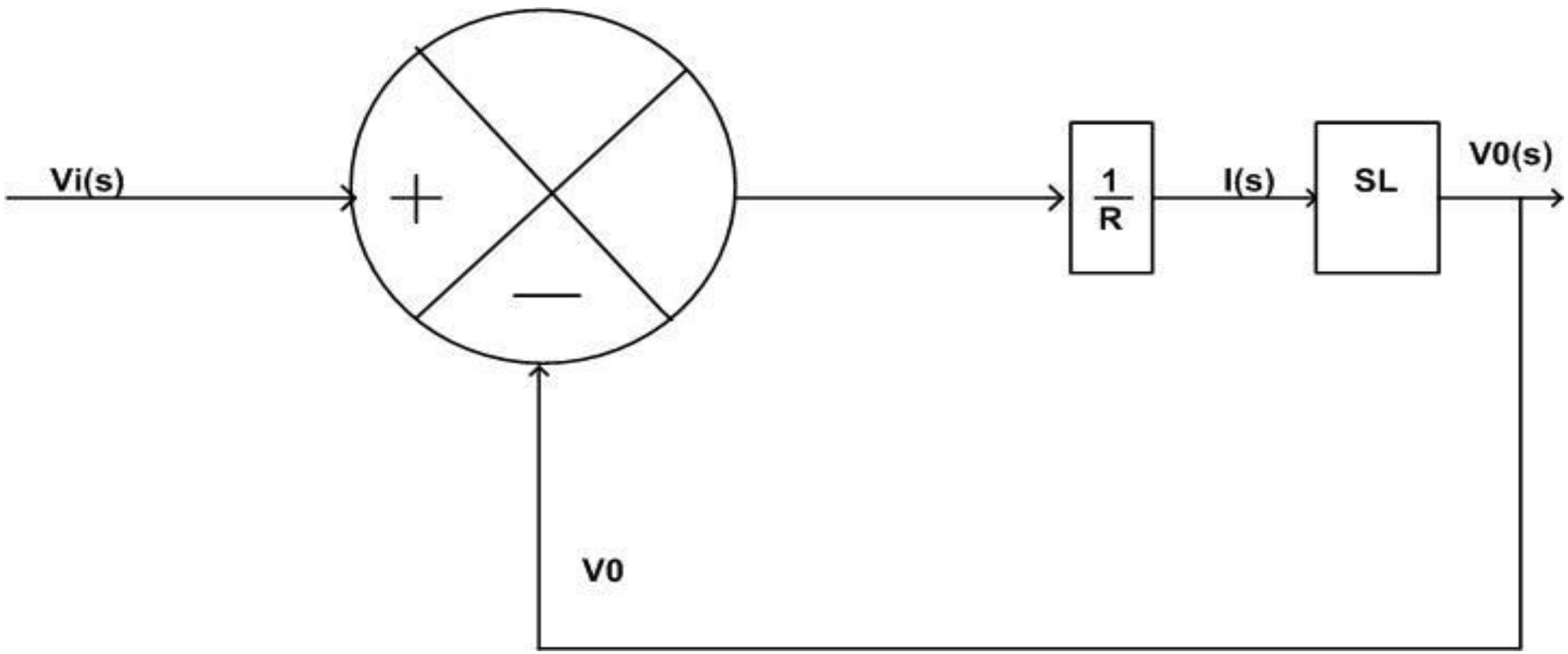
$$I(s) = \frac{1}{R}\left[V_i(s) - V_o(s)\right] --- (8)$$

$$V_o(s) = sLI(s) --- (9)$$

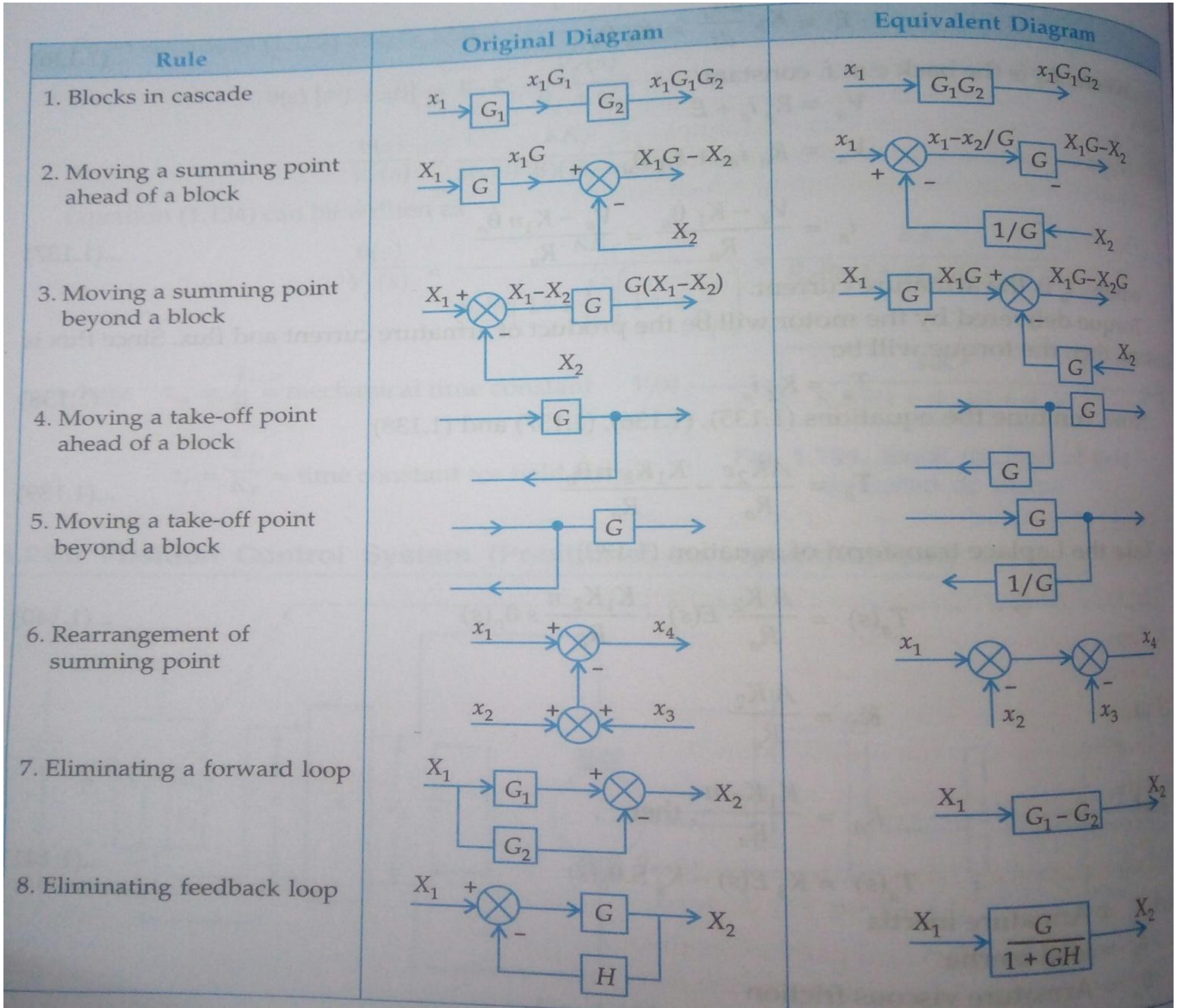

Vi(s)
+
Vi(s)-V0(s)
—
V0(s)


Vi(s)
Vi(s)-Vo(s)
I(s)
+
1/R
-
Vo(s)

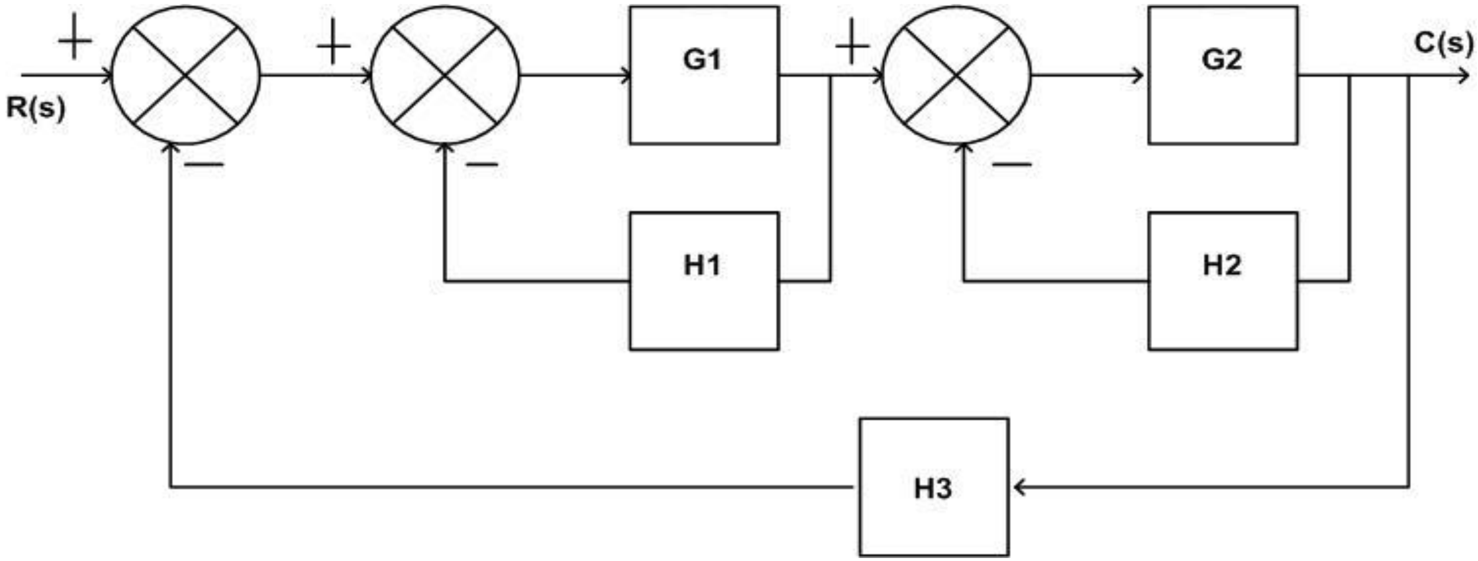

I(s)
SL
V0(s)


Vi(s)
+
—
1
R
I(s)
SL
V0(s)
V0

BLOCK DIAGRAM REDUCTION

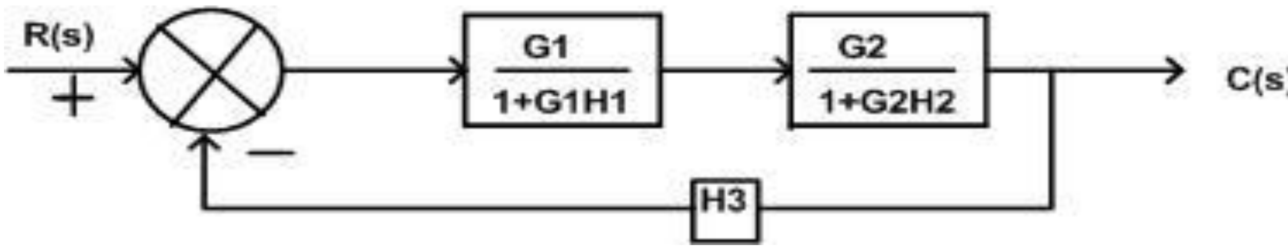


Drive the transfer function using block reduction technique.



Step1: There are two internal closed loop, remove these loops by using closed loop formula

Step 2: Two blocks are cascade



Step 3: Again it is a closed loop

$$R(s) \rightarrow \boxed{\dfrac{G_1G_2}{1+G_1H_1+G_2H_2+G_1H_3G_2+G_1G_2H_1H_2}} \rightarrow C(s)$$

Step 4: required transfer function

$$\frac{C(s)}{R(s)} = \frac{G_1G_2}{1+G_1H_1+G_2H_2+G_1G_2H_3+G_1G_2H_1H_2}$$

THANK YOU

AUTOMATIC CONTROL SYSTEMS

BOOKS

1. CONTROL SYSTEM — B.C. KUO
2. CONTROL SYSTEMS — K. OGATTA
3. CONTROL SYSTEMS — NAGRATH & KOTHARI
4. AUTOMATIC CONTROL SYSTEMS — S.HASAN SAEED

OUTLINE

- Basic definitions.
- Requirement of control systems.
- Classification of control systems.
- Open loop and closed loop control systems.
- Comparison of open loop and closed loop systems.
- Components of closed loop system

INTRODUCTION TO CONTROL SYSTEMS

BASIC DEFINITIONS

SYSTEM : A system is collection of or combination of objects or components connected together in such a manner to attain a certain objective.

OUTPUT : The actual response obtained from a system is called output.

INPUT: The excitation applied to a control system from an external source in order to produce the output is called input.

CONTROL: Control means to regulate, direct or command a system so that the desired objective is obtained.

CONTROL SYSTEM: A control system is a system in which the output quantity is controlled by varying the input quantity.

REQUIREMENT OF A GOOD CONTROL SYSTEM:

(i) Accuracy: Accuracy is high as any error arising should be corrected. Accuracy can be improved by using feedback element. To increase accuracy error detector should be present in control system.

(ii) Sensitivity: Control system should be insensitive to environmental changes, internal disturbances or any other parameters.

(iii) Noise: Noise is an undesirable input signal. Control system should be insensitive to such input signals.

(iv) Stability: In the absence of the inputs, the output should tend to zero as time increases. A good control system should be stable.

(iv) Stability: In the absence of the inputs, the output should tend to zero as time increases. A good control system should be stable.

- (v) Bandwidth: The range of operating frequency decides the bandwidth. For frequency response bandwidth should be large.

- (vi) Speed: The speed of control system should be high.

- (vii) Oscillations: For good control system, oscillation should be constant or sustained oscillation which follows the Barkhausein's criteria.

CLASSIFICATION OF CONTROL SYSTEMS

Depending upon the purpose control system can be classified as follows *(ref. Control systems by A. Anand Kumar)*

1. Depending on hierarchy:

(i) Open loop control systems

(ii) Closed loop control systems

(iii) Adaptive control systems

(iv) Learning control systems

2. Depending on the presence of Human being as apart of the control system

(i) Manually control systems (ii) Automatic control systems

3. Depending on the presence of feedback

(i) Open loop control systems

(ii) Closed loop control systems or feedback control systems.

4. According to the main purpose of the system

(i) Position control systems

(ii) Velocity control systems

(iii) Process control systems

(iv) Temperature control systems

(v) Traffic control systems etc.

Feedback control systems may be classified as

(i) linear control systems and nonlinear control systems.

(ii) Continuous data control systems and discrete data control systems , ac (modulated) control systems and dc (unmodulated) control systems.

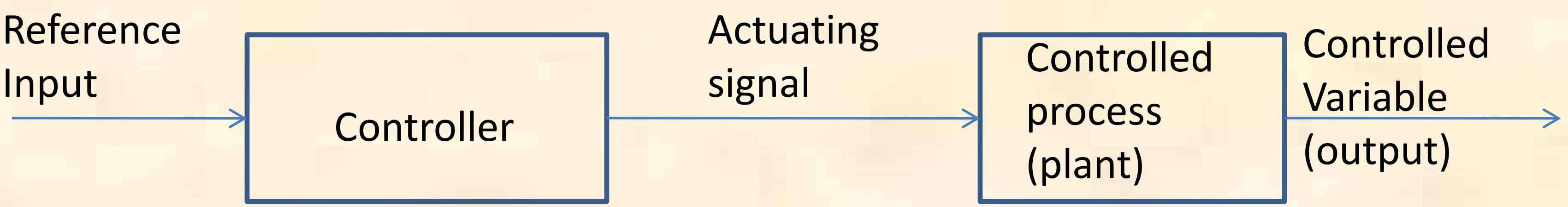
(iii) Multi input multi output(MIMO) system and single input single output (SISO) systems.

(iv) Depending upon the number of poles of the transfer function at the origin, systems may be classified as Type-0, Type-1, Type-2 systems etc.

(v) Depending on the order of the differential equation, control systems may be classified as first order system, second order system etc.

(vi) According to type of damping control systems may be classified as Undamped control systems, damped control systems, Critically damped control systems and Overdamped control systems.

OPEN LOOP CONTROL SYSTEMS: Those systems in which the output has no effect on the control action, i.e. on the input are called open loop control systems. In any open loop control system, the output is not compared with reference input.

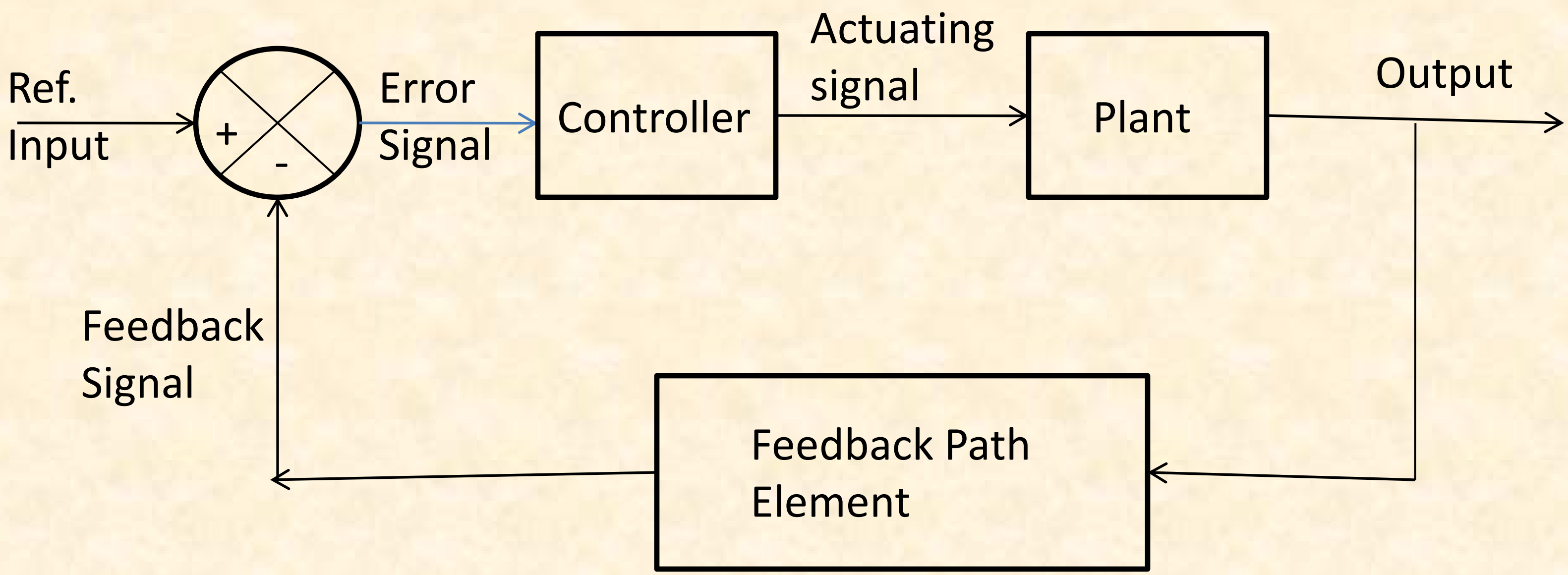


Block Diagram of an open loop system

Example of open loop system is washing machine- soaking, washing and rinsing in the washer operate on a time basis.

CLOSED LOOP CONTROL SYSTEM: Closed loop control systems are also known as feedback control systems. A system that maintains a prescribed relationship between the output and reference input by comparing them and using the difference as a mean of control is called feedback control system.

Example of feedback control system is a room temperature control system. By measuring the room temperature and comparing it with the desired temperature, the thermostat turns the cooling or heating equipment on or off.

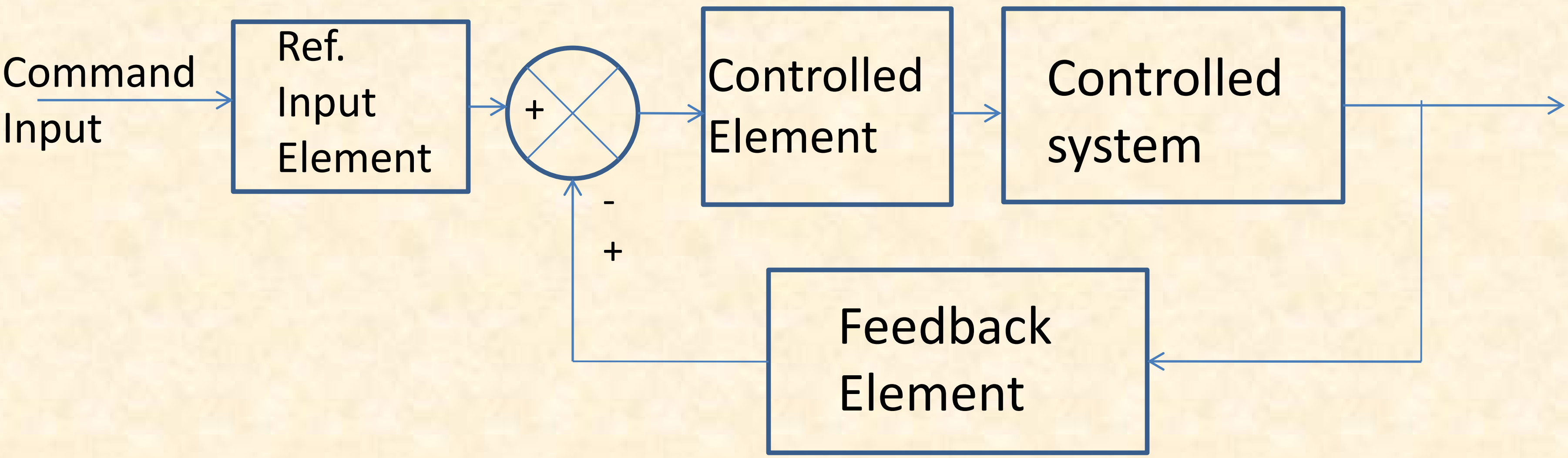


Block Diagram of a Closed Loop System

OPEN LOOP SYSTEM v/s CLOSED LOOP SYSTEM

S.NO.	OPEN LOOP SYSTEM	CLOSED LOOP SYSTEM
1.	These are not reliable	These are reliable
2.	It is easier to build	It is difficult to build
3.	They Consume less power	They Consume more power
4.	They are more stable	These are less stable
5.	Optimization is not possible	Optimization is possible

COMPONENTS OF CLOSED LOOP SYSTEMS



- Command: The command is the externally produced input and independent of the feedback.
- Reference input element: This produces the standard signals proportional to the command.
- Control element: This regulate the output according to the signal obtained from error detector.
- Controlled System: This represents what we are controlling by the feedback loop.
- Feedback Element: This element fed back the output to the error detector for comparison with the reference input.

THANK YOU

FOR

YOUR ATTENTION

SIGNAL FLOW GRAPH (SFG)

OUTLINE

- ✓ Introduction to signal flow graph.
  - *Definitions*
  - *Terminologies*
  - *Properties*
  - *Examples*
- ✓ Mason's Gain Formula.
- ✓ Construction of Signal Flow Graph.
- ✓ Signal Flow Graph from Block Diagram.
- ✓ Block Diagram from Signal Flow Graph.
- ✓ Effect of Feedback.

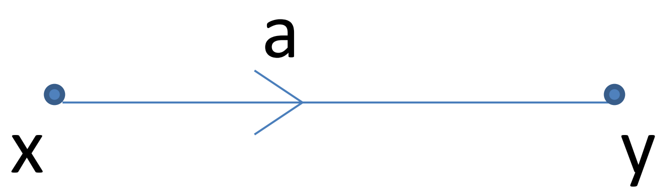
INTRODUCTION

- SFG is a simple method, developed by S.J.Mason
- Signal Flow Graph (SFG) is applicable to the linear systems.
- It is a graphical representation.
- A signal can be transmitted through a branch only in the direction of the arrow.

- Consider a simple equation

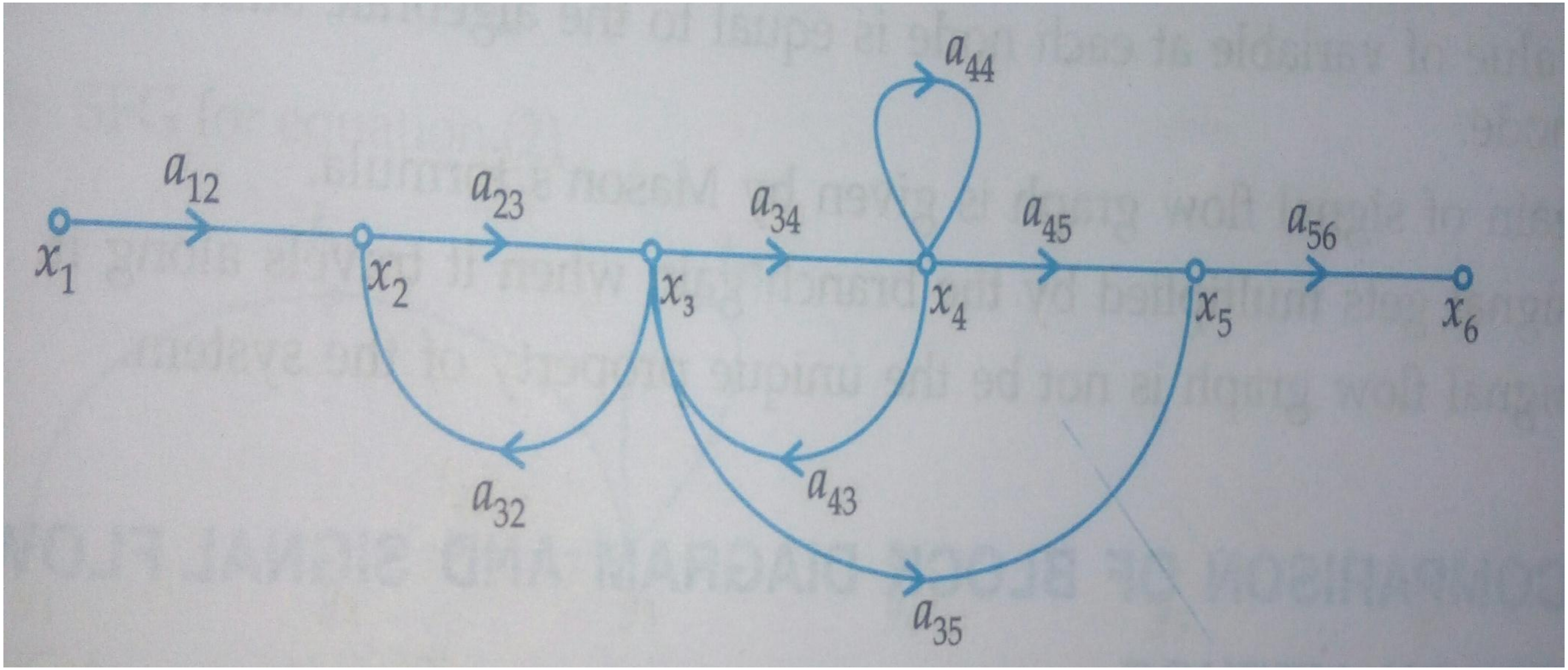
$$y= a\, x$$

- The signal Flow Graph of above equation is shown below



- Each variable in SFG is designed by a *Node.*
- Every transmission function in SFG is designed by a *branch.*
- Branches are *unidirectional.*
- The arrow in the branch denotes the *direction* of the signal flow.

TERMINOLOGIES



- **Input Node or source node:** input node is a node which has only outgoing branches. X_1 is the input node.
- **Output node or sink node:** an output node is a node that has only one or more incoming branches. X_6 is the output node.
- **Mixed nodes:** a node having incoming and outgoing branches is known as mixed nodes. x_2, x_3, x_4 and x_5 are the mixed nodes.
- **Transmittance:** Transmittance also known as transfer function, which is normally written on the branch near arrow.

- **Forward path:** Forward path is a path which originates from the input node and terminates at the output node and along which no node is traversed more than once. X_1-x_2-x_3-x_4-x_5-x_6 and x_1-x_2-x_3-x_5-x_6 are forward paths.
- **Loop:** loop is a path that originates and terminates on the same node and along which no other node is traversed more than once. e.g x_2 to x_3 to x_2 and x_3 to x_4 to x_3 .
- **Self loop:** it is a path which originates and terminates on the same node. For example x_4

- **Path gain:** The product of the branch gains along the path is called path gain. For example the gain of the path x_1-x_2-x_3-x_4-x_5-x_6 is $a_{12}a_{23}a_{34}a_{45}a_{56}$
- **Loop Gain:** The gain of the loop is known as loop gain.
- **Non-touching loop:** Non-touching loops having no common nodes branch and paths. For example x_2 to x_3 to x_2 and x_4 to x_4 are non-touching loops.

PROPERTIES OF SIGNAL FLOW GRAPH:

1. Signal flow graph is applicable only for linear time-invariant systems.
2. The signal flow graph along the direction of the arrow.
3. The gain of the SFG is given by Mason's formula.
4. The signal gets multiplied by branch gain when it travels along it.
5. The SFG is not be the unique property of the sytem.
6. Signal travel along the branches only in the direction described by the arrows of the branches.

COMPARISON OF BLOCK DIAGRAM AND SIGNAL FLOW GRAPH METHOD

S.NO.	BLOCK DIAGRAM	SFG
1.	Applicable to LTI systems only	Applicable to LTI systems only
2.	Each element is represented by block	Each variable is represented by node
3.	Summing point and take off points are separate	Summing point and take off points are not used
4.	Self loop do not exists	Self loop can exits
5.	It is time consuming method	Requires less time
6.	Feedback path is present	Feedback loops are present.

CONSTRUCTION OF SFG FROM EQUATIONS

Consider the following equations

$$y_2 = t_{21}y_1 + t_{23}y_3$$

$$y_3 = t_{32}y_2 + t_{33}y_3 + t_{31}y_1$$

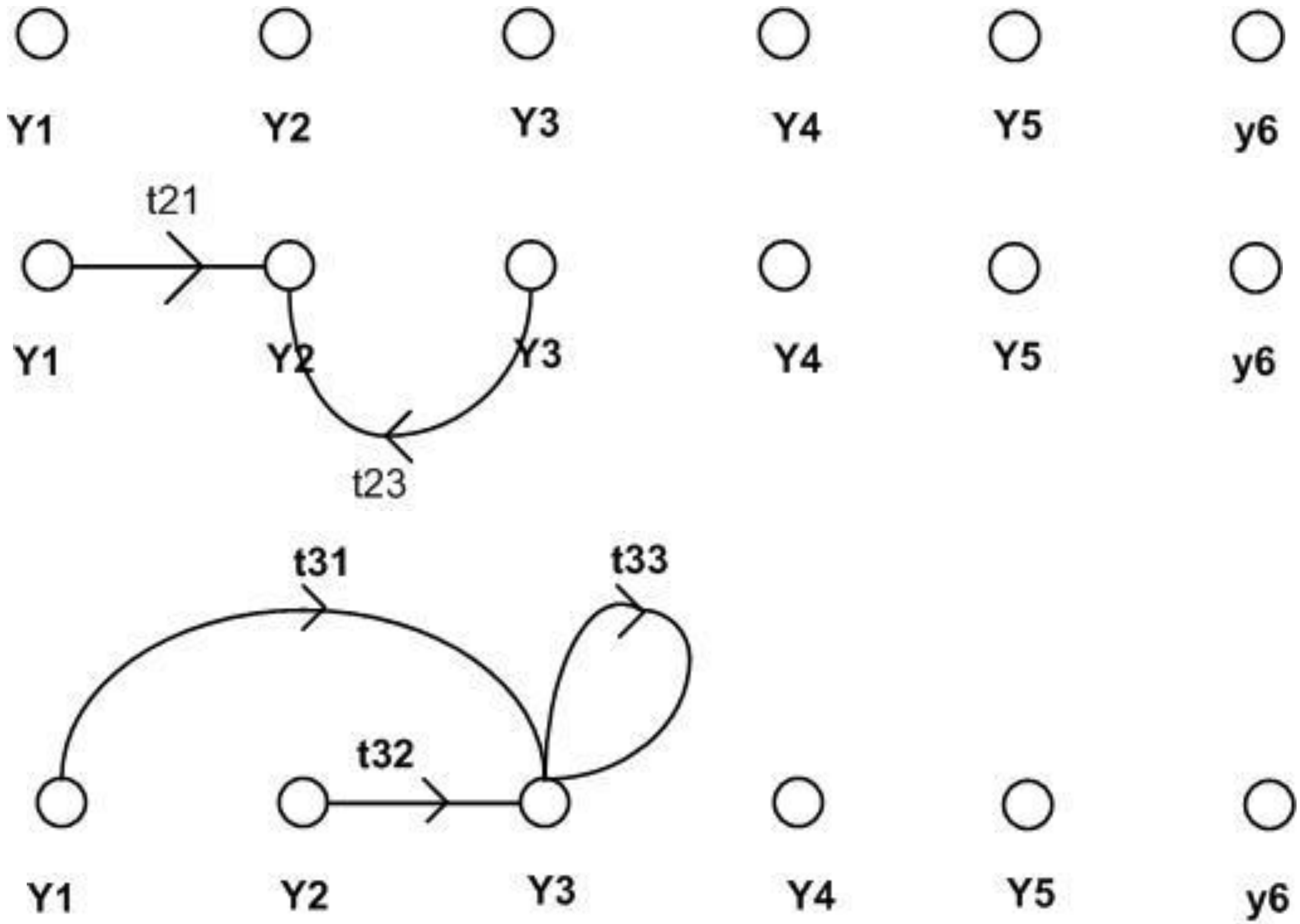
$$y_4 = t_{43}y_3 + t_{42}y_2$$

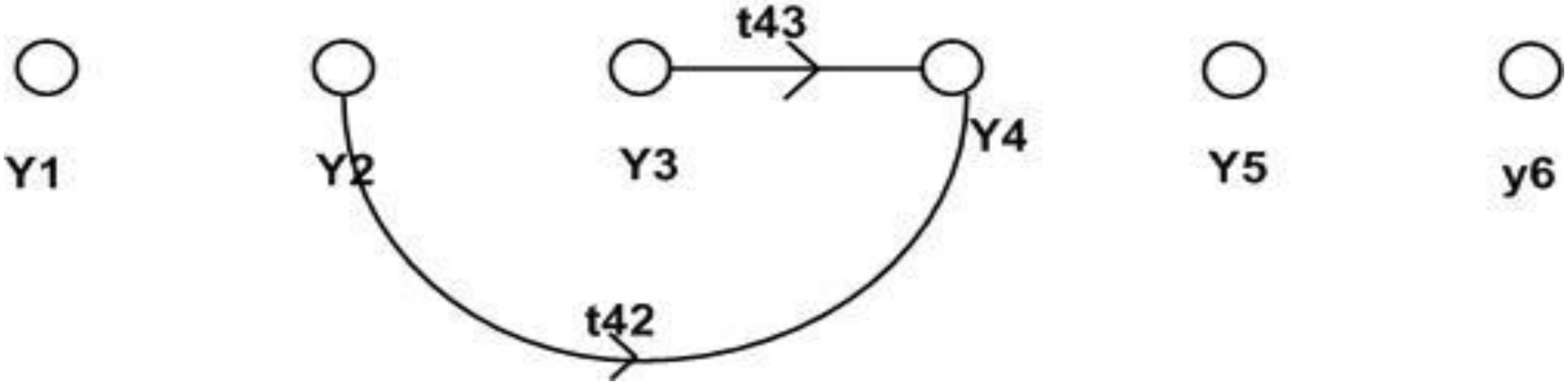
$$y_5 = t_{54}y_4$$

$$y_6 = t_{65}y_5 + t_{64}y_4$$

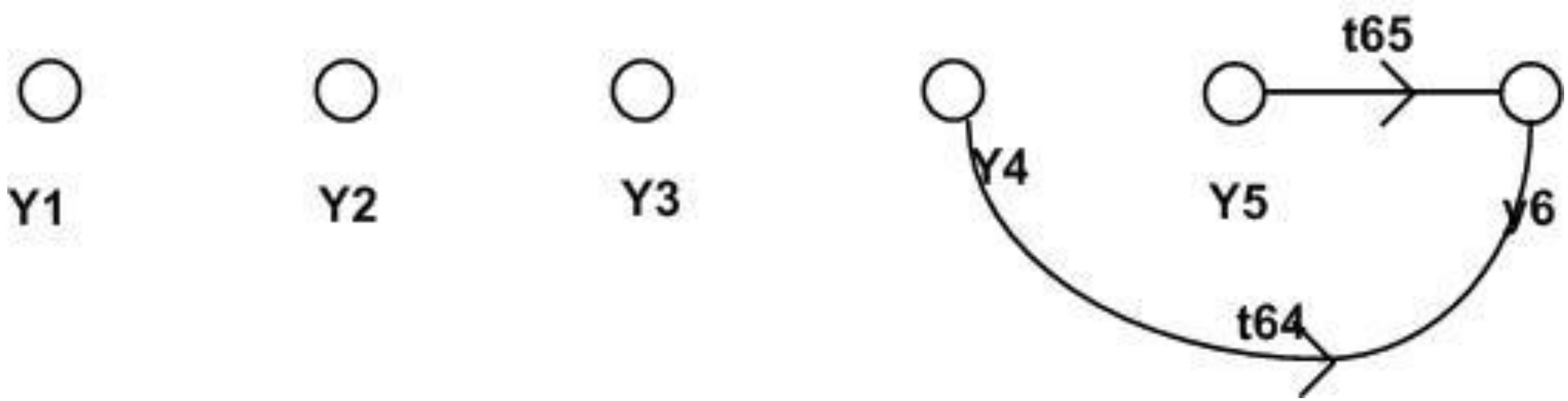
Where y_1 is the input and y_6 is the output

First draw all nodes. In given example there are six nodes. From the first equation it is clear that y_2 is the sum of two signals and so on.

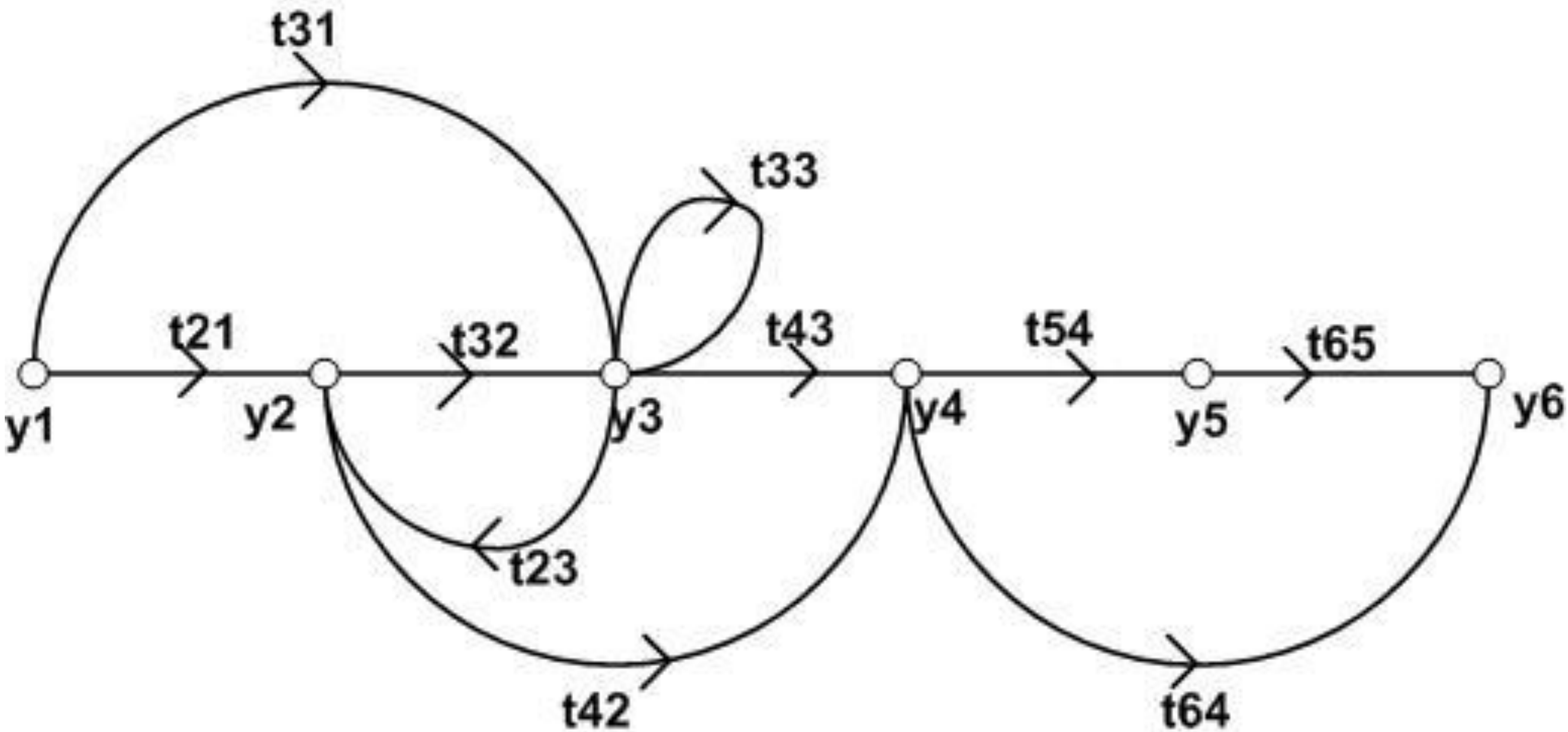

Y1
Y2
Y3
Y4
Y5
y6
t21
Y1
Y2
Y3
Y4
Y5
y6
t23
t31
t33
t32
Y1
Y2
Y3
Y4
Y5
y6


t43
Y1
Y2
Y3
Y4
Y5
y6
t42


t54
Y1
Y2
Y3
Y4
Y5
y6

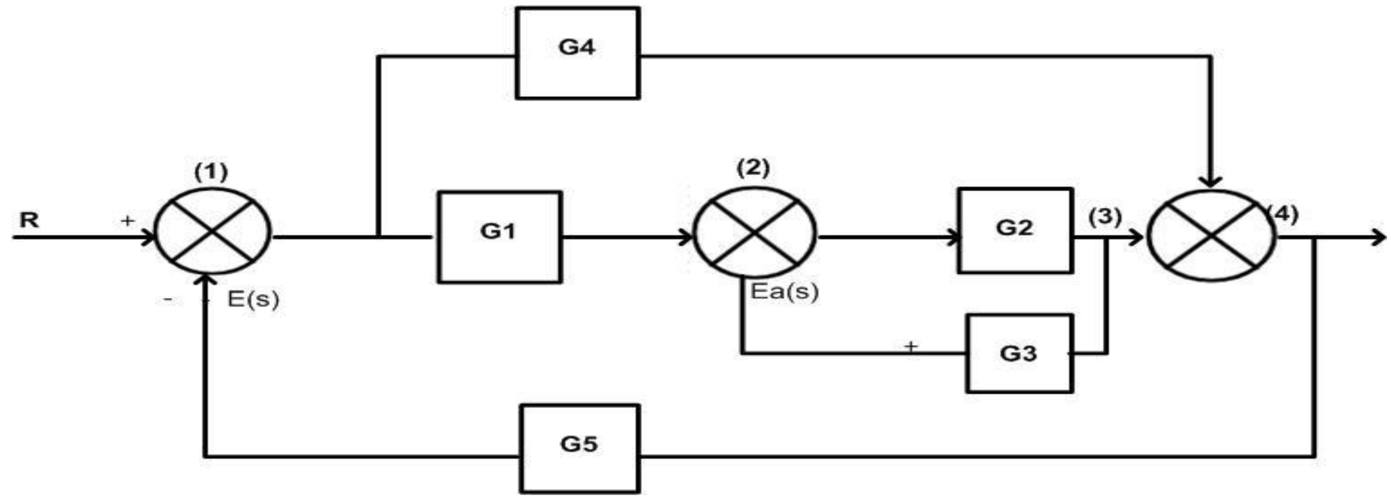

t65
Y1
Y2
Y3
Y4
Y5
y6
t64

Complete Signal Flow Graph

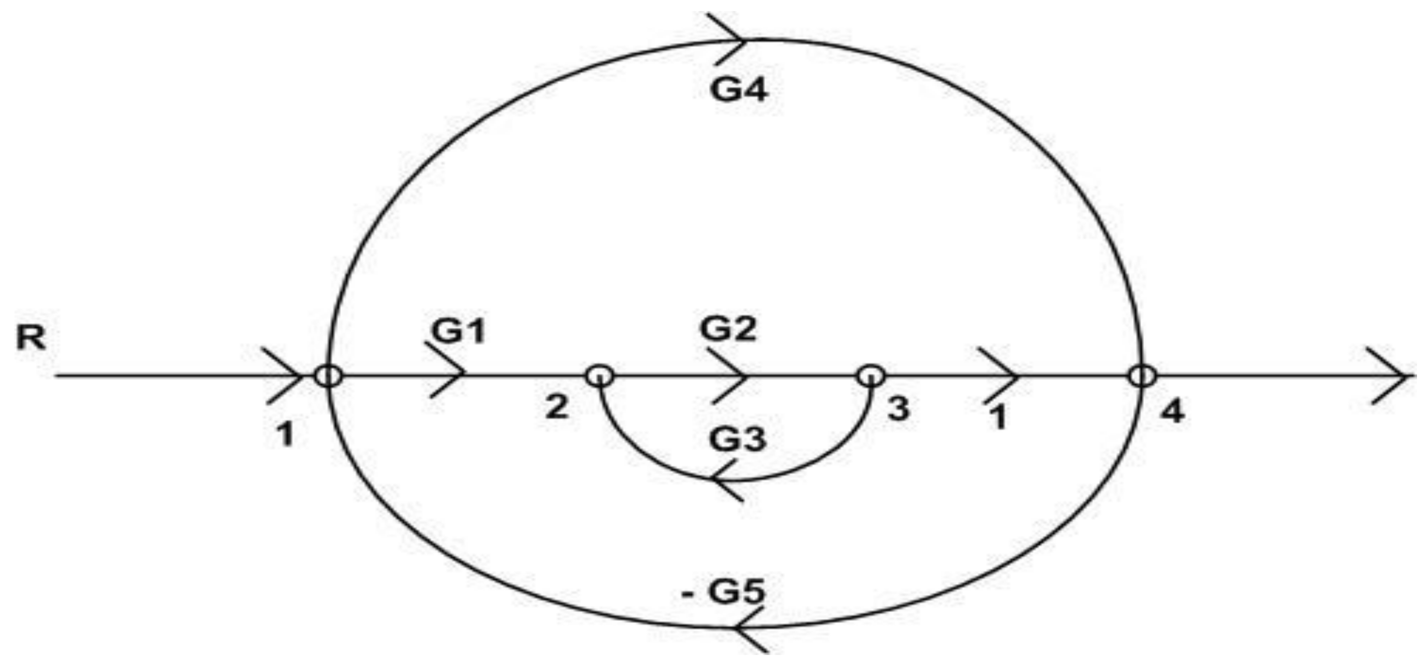


CONSTRUCTION OF SFG FROM BLOCK DIAGRAM

- All variables, summing points and take off points are represented by nodes.
- If a summing point is placed before a take off point in the direction of signal flow, in such case represent the summing point and take off point by a single node.
- If a summing point is placed after a take off point in the direction of signal flow, in such case represent the summing point and take off point by separate nodes by a branch having transmittance unity.

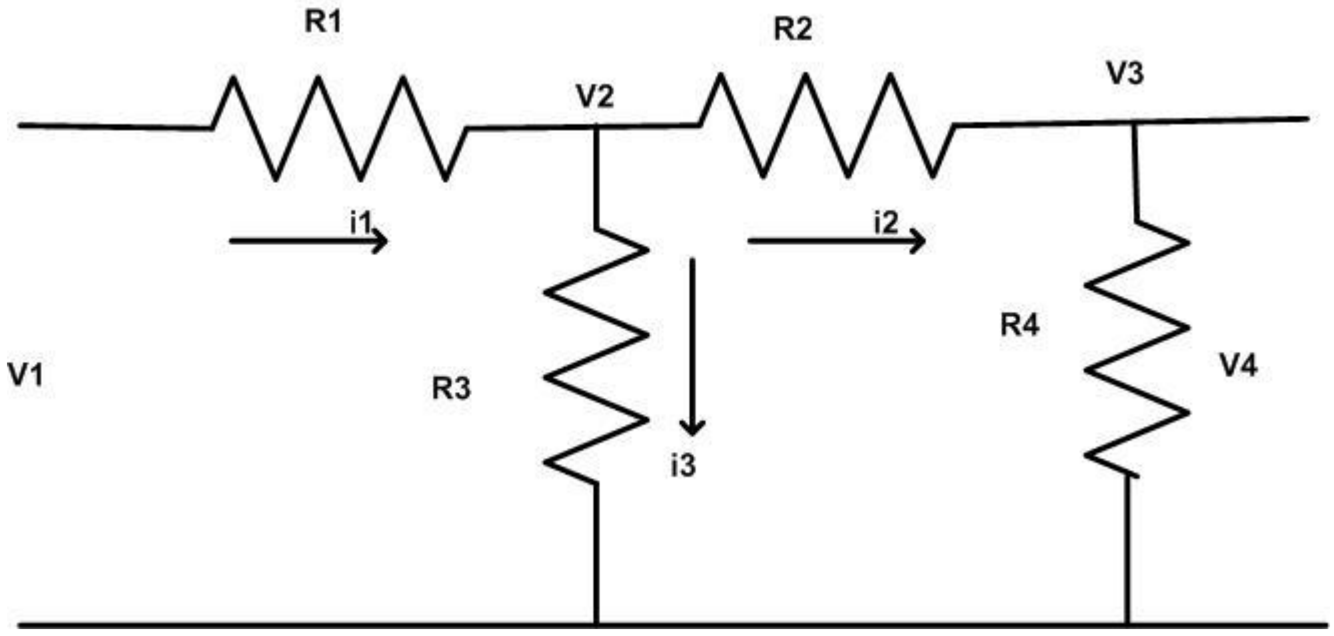


G4
(1)
(2)
R
+
G1
G2
(3)
(4)
-
E(s)
Ea(s)
+
G3
G5



G4
R
G1
G2
1
2
3
1
4
G3
- G5

Draw the SFG for the network shown in fig, take V_3 as output node

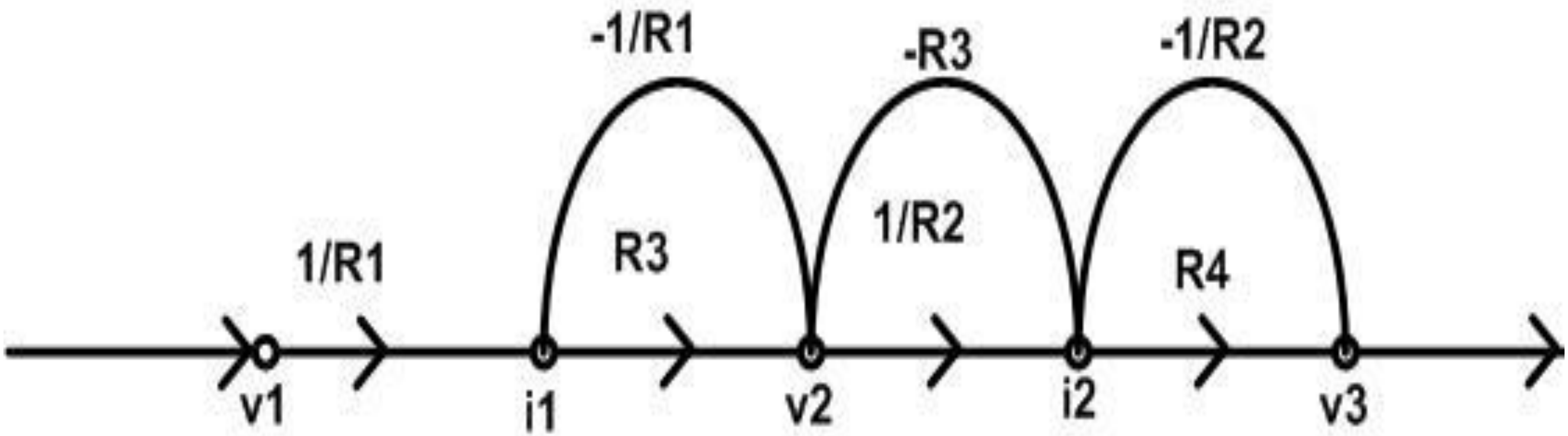


$$i_1 = \frac{v_1}{R_1} - \frac{v_2}{R_1}$$

$$v_2 = i_3 R_3 = R_3(i_1 - i_2) = R_3 i_1 - R_3 i_2$$

$$i_2 = \frac{v_2}{R_2} - \frac{v_3}{R_2}$$

$$v_3 = R_4 i_2$$



MASON'S GAIN FORMULA

By using Mason's Gain formula we can determine the overall transfer function of the system in one step.

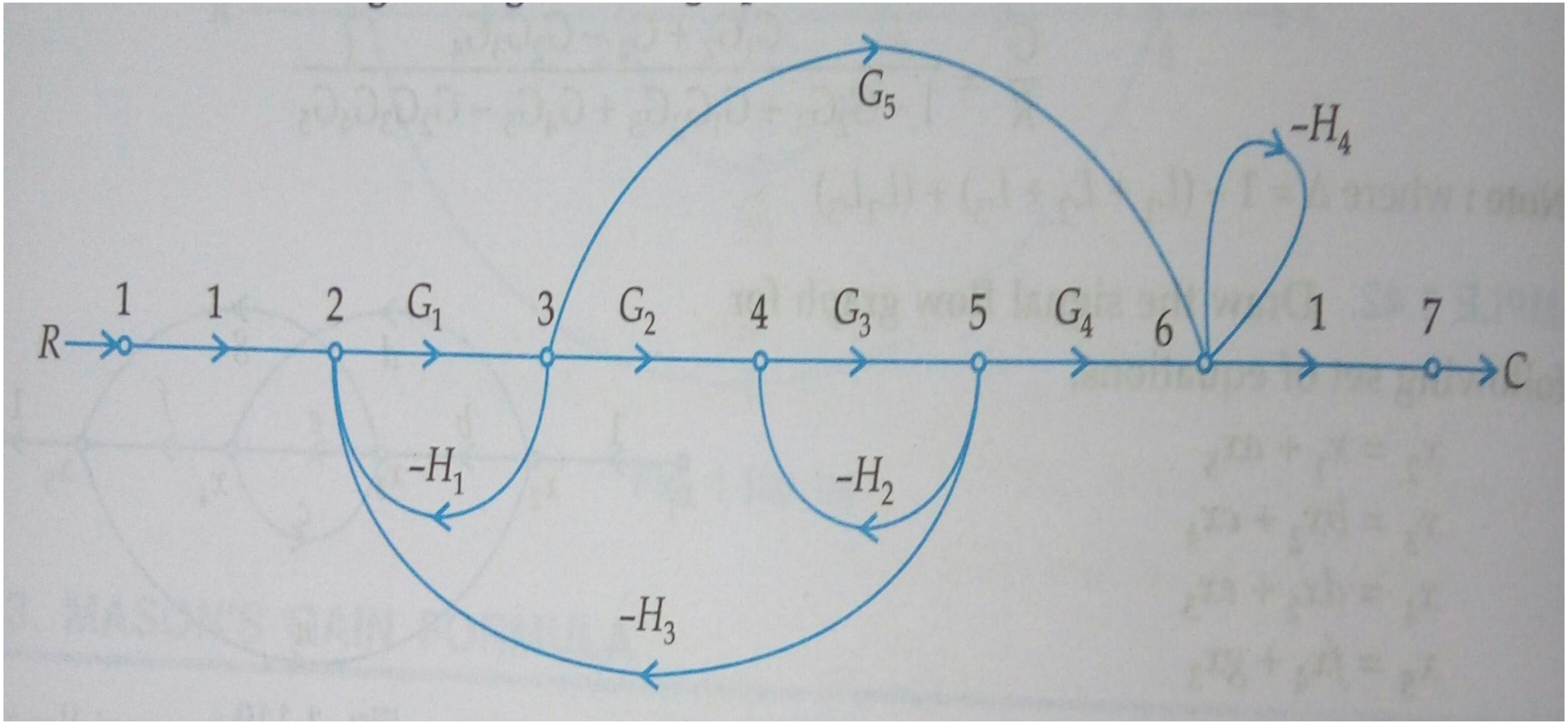
$$T = \frac{\sum g_k \Delta_k}{\Delta}$$

Where Δ=1- [sum of all individual loop gain]+[sum of all possible gain products of two non-touching loops]-[sum of all possible gain products of three non-touching loops]+...........

g_k = gain of k^{th} forward path

Δ_k =the part of Δ not touching the k^{th} forward path

For the given Signal Flow Graph find the ratio C/R by using Mason's Gain Formula.



The gain of the forward path $g_1 = G_1G_2G_3G_4$

$$g_2 = G_1G_5$$

Individual loops

$$L_1 = -G_1H_1$$

$$L_2 = -G_3H_2$$

$$L_3 = -G_1G_2G_3H_3$$

$$L_4 = -H_4$$

Two non-touching loops

$$L_1L_2 = G_1H_1G_3H_2$$

$$L_1L_4 = G_1H_1H_4$$

$$L_2L_4 = G_3H_2H_4$$

$$L_3L_4 = G_1G_2G_3H_3H_4$$

Three non-touching loops

$$L_1L_2L_4 = -G_1H_1G_3H_2H_4$$

$$\Delta_1 = 1-0$$

$$\Delta_2 = 1+G_3H_2$$

$$\Delta = 1-(L_1+L_2+L_3+L_4)+(L_1L_2+L_1L_4+L_2L_4+L_3L_4)-(L_1L_2L_4)$$

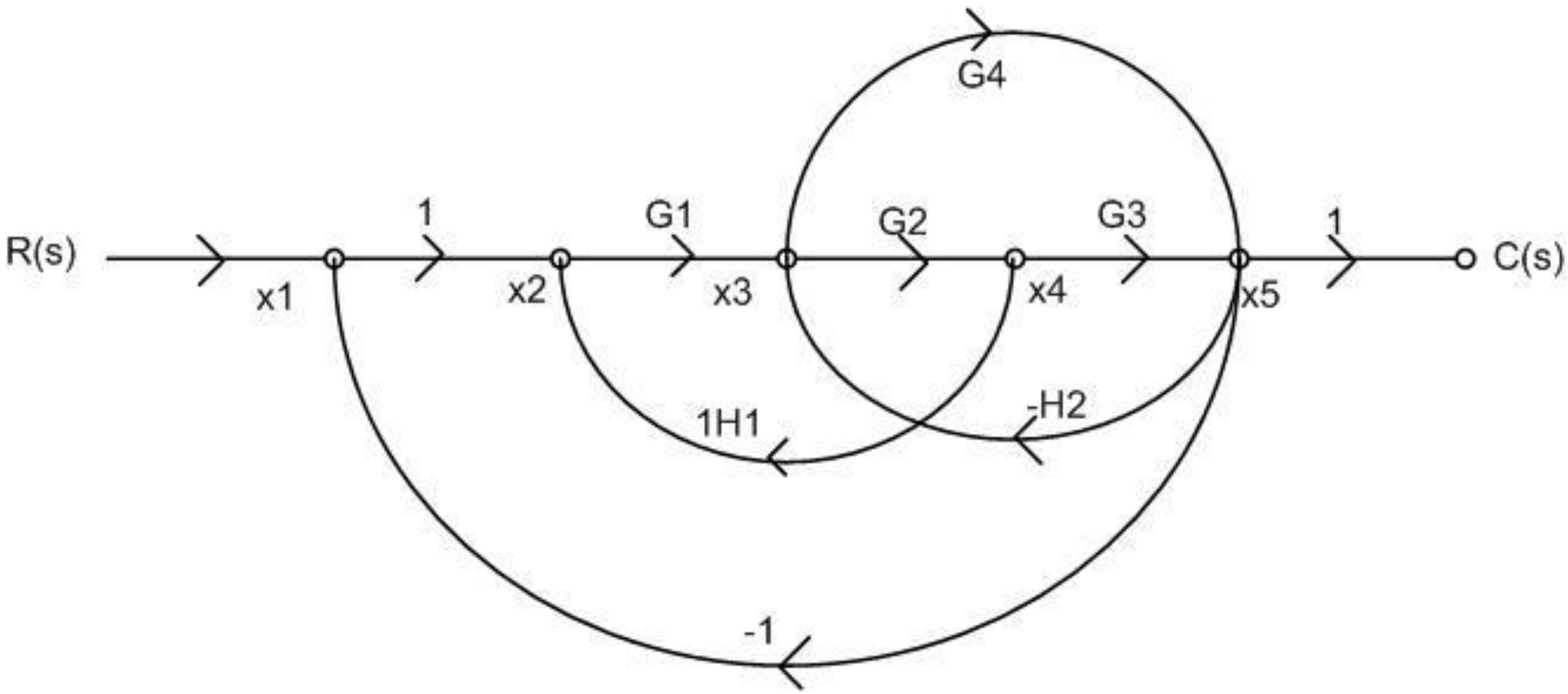
$$\frac{C}{R} = \frac{g_1\Delta_1+g_2\Delta_2}{\Delta}$$

$$\frac{C}{R} = \frac{G_1G_2G_3G_4+G_1G_5(1+G_3H_2)}{1+G_1H_1+G_3H_2+G_1G_2G_3H_3+H_4+G_1G_3H_1H_2+ G_1H_1H_4+G_3H_2H_4+G_1G_2G_3H_3H_4+G_1G_3H_1H_2H_4}$$

BLOCK DIAGRAM FROM SFG

- For given SFG, write the system equations.
- At each node consider the incoming branches only.
- Add all incoming signals algebraically at a node.
- for + or – sign in system equations use a summing point.
- For the gain of each branch of signal flow graph draw the block having the same transfer function as the gain of the branch.

Draw the block diagram from the given signal flow graph



Solution:

- At node x_1 the incoming branches are from R(s) and x_5

$$x_1 = 1.R(s) - 1\ x_5$$

- At node $x_{2,}$ there are two incoming branches

$$x_2 = 1\ x_1 - H_1 x_4$$

- At node x_3 there are two incoming branches

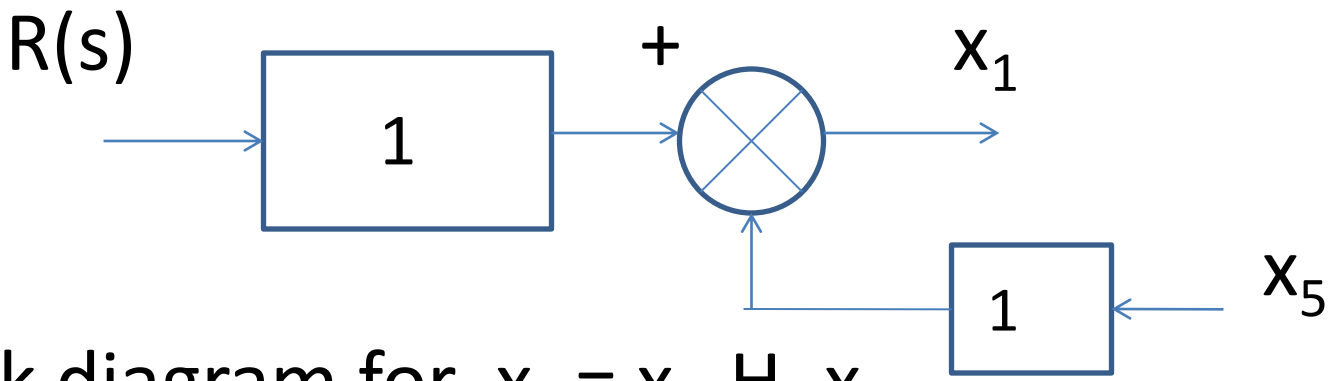
$$X_3 = G_1 x_2 - H_2 x_5$$

- Similarly at node x_4 and x_5 the system equations are

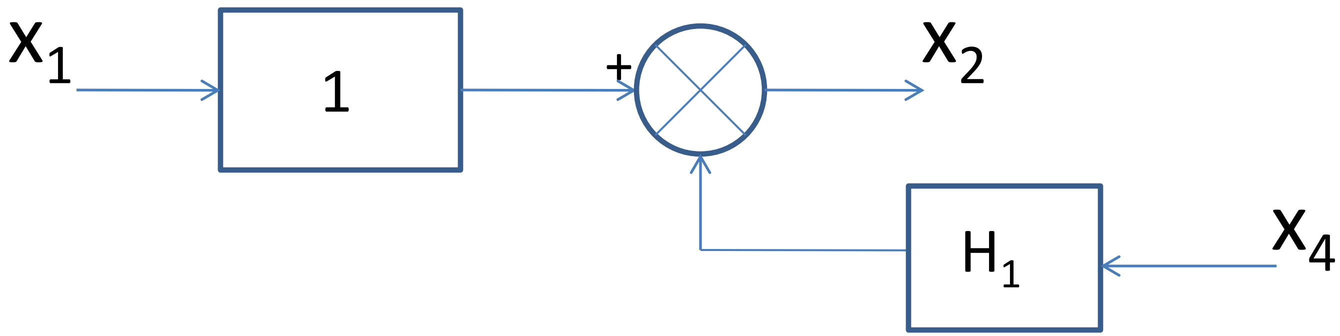
$$X_4 = G_2\ x_3$$

$$x_5 = G_3 x_4 + G_4 x_3$$

Draw the block diagram for x_1=1.R(s)-1 x_5



Block diagram for $x_2 = x_1 - H_1 x_4$



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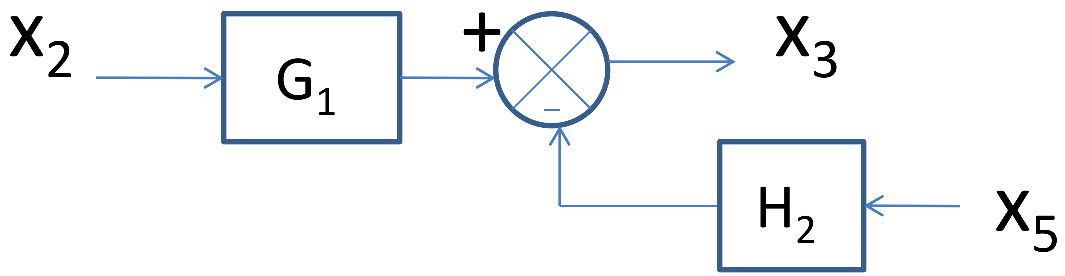
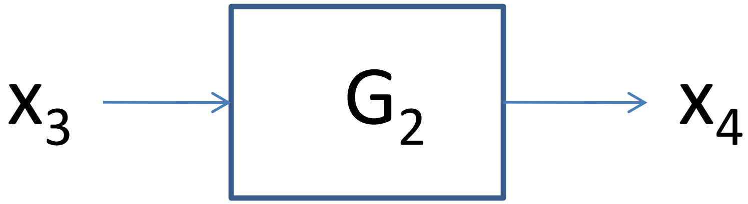
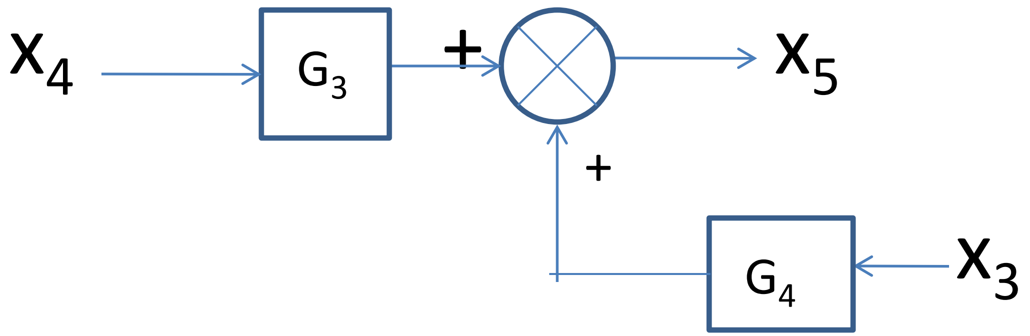
Block diagram for $x_3 = G_1 x_2 - H_2 x_5$



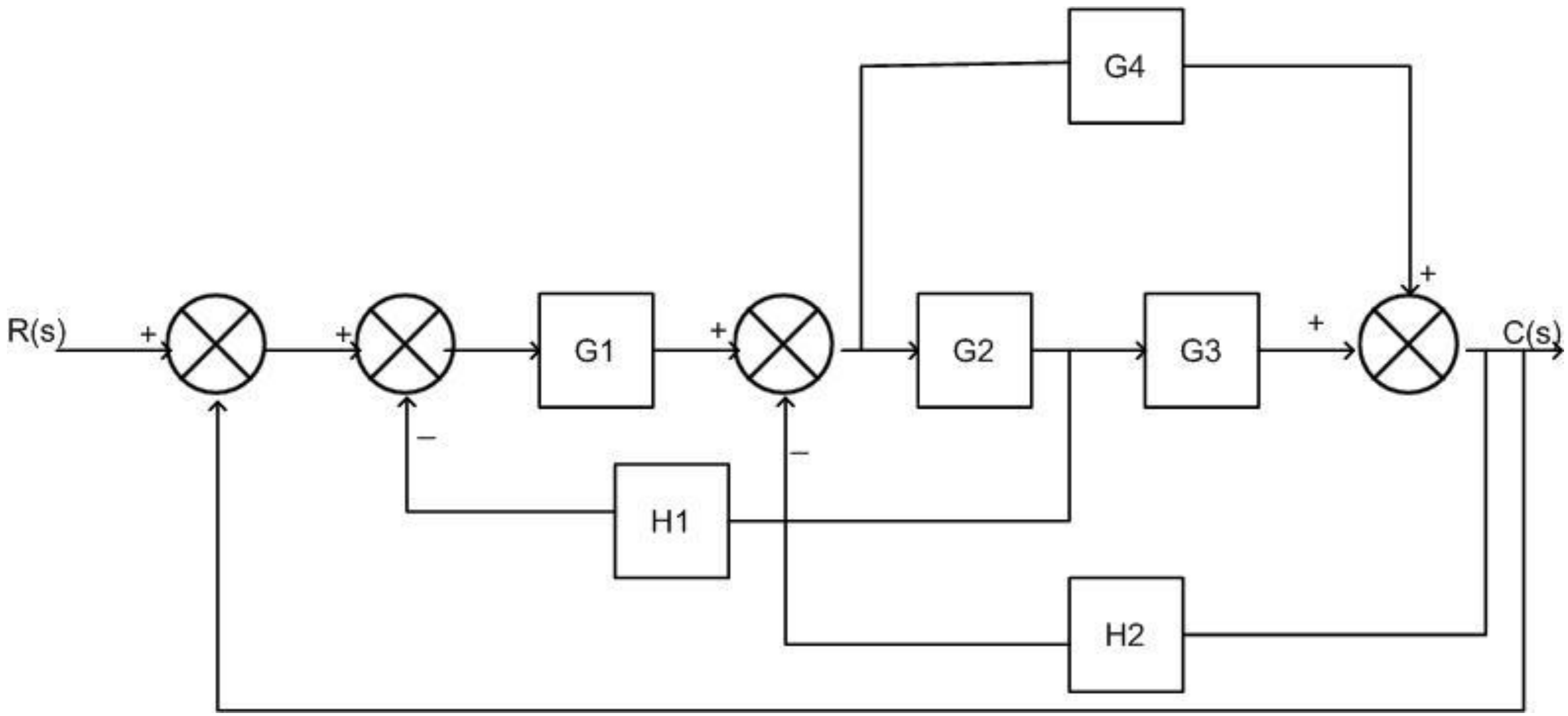
Block diagram for $x_4 = G_2 x_3$



Block diagram for $x_5 = G_3 x_4 + G_4 x_3$



Combining all block diagram



EFFECT OF FEEDBACK ON OVERALL GAIN

$$R(s) \longrightarrow \boxed{G(s)} \longrightarrow C(s)$$

The overall transfer function of open loop system is

$$\frac{C(s)}{R(s)} = G(s)$$

The overall transfer function of closed loop system is

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

For negative feedback the gain G(s) is reduced by a factor

$$\frac{1}{1+G(s)H(s)}$$

So due to negative feedback overall gain of the system is reduced

EFFECT OF FEEDBACK ON STABILITY

Consider the open loop system with overall transfer function

$$G(s) = \frac{K}{s+T}$$

The pole is located at s=-T

Now, consider closed loop system with unity negative feedback, then overall transfer function is given by

$$\frac{C(s)}{R(s)} = \frac{K}{s+(T+K)}$$

Now closed loop pole is located at s=-(T+K)

Thus, feedback controls the time response by adjusting the location of poles. The stability depends upon the location of the poles. Thus feedback affects the stability.

OUTLINE

- Definition of Transfer Function.
- Poles & Zeros.
- Characteristic Equation.
- Advantages of Transfer Function.
- Mechanical System (i) Translational System and (ii) Rotational System.
- Free Body diagram.
- Transfer Function of Electrical, Mechanical Systems.
- D'Alembert Principle.

TRANSFER FUNCTION

Definition: The transfer function is defined as the ratio of Laplace transform of the output to the Laplace transform of input with all initial conditions are zero.

We can defined the transfer function as

$$\frac{C(s)}{R(s)} = G(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \ldots.. + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \ldots.. + a_1 s + a_0} ----------(1.0)$$

In equation (1.0), if the order of the denominator polynomial is greater than the order of the numerator polynomial then the transfer function is said to be STRICTLY PROPER. If the order of both polynomials are same, then the transfer function is PROPER. The transfer function is said to be IMPROPER, if the order of numerator polynomial is greater than the order of the denominator polynomial.

CHARACTERISTIC EQUATION: The characteristic equation can be obtained by equating the denominator polynomial of the transfer function to zero. That is

$$a_n s^n + a_{n-1} s^{n-1} + \ldots\ldots + a_1 s + a_0 = 0$$

POLES AND ZEROS OF A TRANSFER FUNCTION

POLES : The poles of G(s) are those values of 's' which make G(s) tend to infinity.

ZEROS: The zeros of G(s) are those values of 's' which make G(s) tend to zero.

If either poles or zeros coincide, then such type of poles or zeros are called multiple poles or zeros, otherwise they are known as simple poles or zeros.

For example, consider following transfer function

$$G(s) = \frac{50(s+3)}{s(s+2)(s+4)^2}$$

This transfer function having the simple poles at s=0, s=-2, multiple poles at s=-4 and simple zero at s=-3.

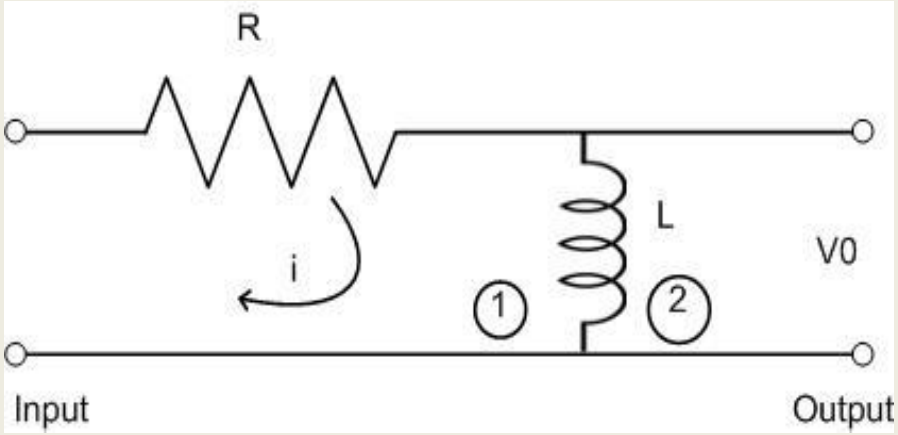
Advantages of Transfer Function:

1. Transfer function is a mathematical model of all system components and hence of the overall system and therefore it gives the gain of the system.
2. Since Laplace transform is used, it converts time domain equations to simple algebraic equations.
3. With the help of transfer function, poles, zeros and characteristic equation can be determine.
4. The differential equation of the system can be obtained by replacing 's' with d/dt.

DISADVANTAGES OF TRANSFER FUNCTION:

1. Transfer function cannot be defined for non-linear system.
2. From the Transfer function , physical structure of a system cannot determine.
3. Initial conditions loose their importance.

Find the transfer function of the given figure.



Solution:

Step 1: Apply KVL in mesh 1 and mesh 2

$$v_i = Ri + L\frac{di}{dt} ----(1)$$

$$v_o = L\frac{di}{dt} ----(2)$$

Step 2: take Laplace transform of eq. (1) and (2)

$$V_i(s) = RI(s) + sLI(s) ------(3)$$

$$V_o(s) = sLI(s) ------(4)$$

Step 3: calculation of transfer function

$$\frac{V_o(s)}{V_i(s)} = \frac{sLI(s)}{(R+sL)I(s)}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{sL}{R+sL} ------(5)$$

Equation (5) is the required transfer function

A system having input x(t) and output y(t) is represented by Equation (1). Find the transfer function of the system.

$$\frac{dy(t)}{dt}+4y(t)=\frac{dx(t)}{dt}+5x(t)------(1)$$

Solution: taking Laplace transform of equation (1)

$$sY(s)+4Y(s)=sX(s)+5X(s)$$

$$Y(s)(s+4)=X(s)(s+5)$$

$$\frac{Y(s)}{X(s)}=\frac{s+5}{s+4}$$

$$G(s)=\frac{Y(s)}{X(s)}=\frac{s+5}{s+4}$$

G(s) is the required transfer function

The transfer function of the given system is given by

$$G(s)=\frac{4s+1}{s^2+2s+3}$$

Find the differential equation of the system having input x(t) and output y(t).

Solution: $$G(s)=\frac{Y(s)}{X(s)}=\frac{4s+1}{s^2+2s+3}$$

$$X(s)[4s+1]=Y(s)[s^2+2s+3]$$

$$4sX(s)+X(s)=s^2+2sY(s)+3Y(s)$$

Taking inverse Laplace transform, we have

$$4\frac{dx(t)}{dt}+x(t)=\frac{dy^2}{dt^2}+2\frac{dy(t)}{dt}+3y(t)$$

Required differential equation is

$$\frac{d^2y(t)}{dt^2}+2\frac{dy(t)}{dt}+3y(t)=4\frac{dx(t)}{dt}+x(t)$$

MECHANICAL SYSTEM

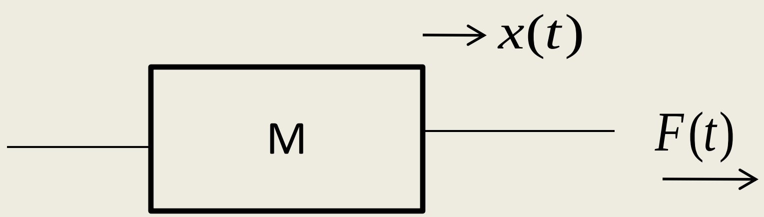
TRANSLATIONAL SYSTEM: The motion takes place along a strong line is known as translational motion. There are three types of forces that resists motion.

INERTIA FORCE: consider a body of mass 'M' and acceleration 'a', then according to Newton's law of motion

$$F_M(t)=Ma(t)$$

If v(t) is the velocity and x(t) is the displacement then

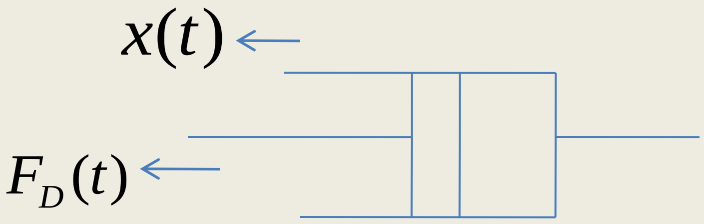
$$F_M(t) = M\frac{dv(t)}{dt} = M\frac{d^2x(t)}{dt^2}$$



DAMPING FORCE: For viscous friction we assume that the damping force is proportional to the velocity.

$$F_D(t)=B\,v(t)=B\frac{dx(t)}{dt}$$

Where B= Damping Coefficient in N/m/sec.



We can represent 'B' by a dashpot consists of piston and cylinder.

SPRING FORCE: A spring stores the potential energy. The restoring force of a spring is proportional to the displacement.

$$F_K(t)=\alpha x(t)=K\,x(t)$$

$$F_K(t)=K\int v(t)\,dt$$

Where ‘K’ is the spring constant or stiffness (N/m)

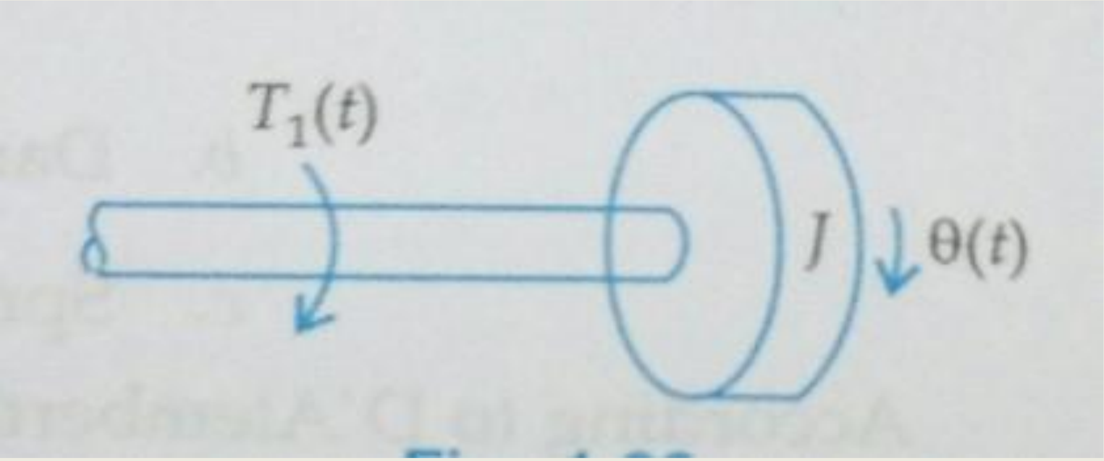
The stiffness of the spring can be defined as restoring force per unit displacement

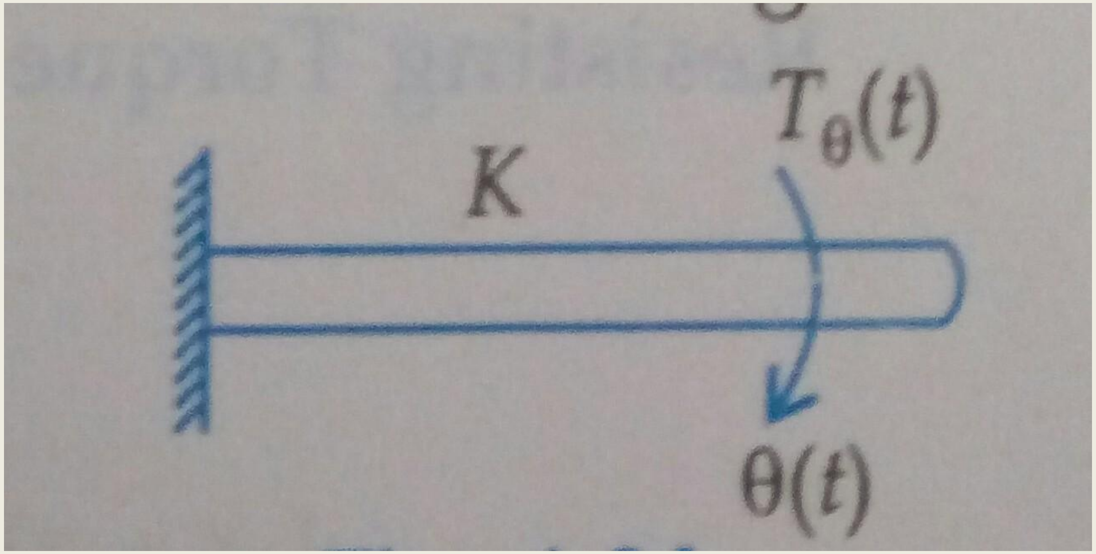
ROTATIONAL SYSTEM: The rotational motion of a body can be defined as the motion of a body about a fixed axis. There are three types of torques resists the rotational motion.

1. Inertia Torque: Inertia(J) is the property of an element that stores the kinetic energy of rotational motion. The inertia torque TI is the product of moment of inertia J and angular acceleration α(t).

$$T_I(t) = J\alpha(t) = J\frac{d\omega(t)}{dt} = J\frac{d^2\theta(t)}{dt^2}$$

Where ω(t) is the angular velocity and θ(t) is the angular displacement.


$T_1(t)$
J
$\theta(t)$


$T_\theta(t)$
K
$\theta(t)$

2. Damping torque: The damping torque $T_D(t)$ is the product of damping coefficient B and angular velocity ω. Mathematically

$$T_D(t) = B\omega(t) = B\frac{d\theta(t)}{dt}$$

3. Spring torque: Spring torque $T_\theta(t)$ is the product of torsional stiffness and angular displacement.

Unit of ‘K’ is N-m/rad

$$T_\theta(t) = K\theta(t)$$

D’ALEMBERT PRINCIPLE

This principle states that “for any body, the algebraic sum of externally applied forces and the forces resisting motion in any given direction is zero”

D'ALEMBERT PRINCIPLE contd……

External Force: F(t)

Resisting Forces :

1. Inertia Force:

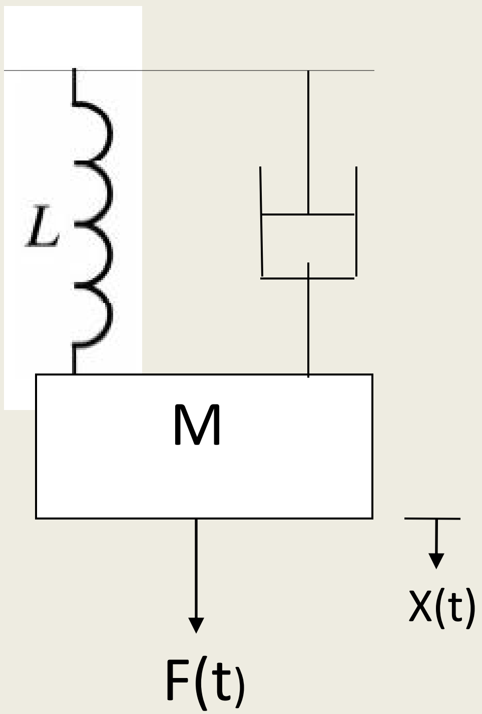
$$F_M(t) = -M\frac{d^2x(t)}{dt^2}$$

2. Damping Force:

$$F_D(t) = -B\frac{dx(t)}{dt}$$

3. Spring Force:

$$F_K(t) = -Kx(t)$$



According to D'Alembert Principle

$$F(t) - M\frac{d^2x(t)}{dt^2} - B\frac{dx(t)}{dt} - Kx(t) = 0$$

$$F(t) = M\frac{d^2x(t)}{dt^2} + B\frac{dx(t)}{dt} + Kx(t)$$

Consider rotational system:

External torque: T(t)

Resisting Torque:

(i) Inertia Torque: $T_I(t) = -J\dfrac{d\omega(t)}{dt}$

(ii) Damping Torque: $T_D(t) = -B\dfrac{d\theta(t)}{dt}$

(iii) Spring Torque: $T_K(t) = -K\theta(t)$

According to D'Alembert Principle:

$$T(t) + T_I(t) + T_D(t) + T_K(t) = 0$$

$$T(t) - J\frac{d\omega(t)}{dt} - B\frac{d\theta(t)}{dt} - K\theta(t) = 0$$

$$T(t) = J\frac{d\omega(t)}{dt} + B\frac{d\theta(t)}{dt} + K\theta(t)$$

D'Alembert Principle for rotational motion states that

"For anybody, the algebraic sum of externally applied torques and the torque resisting rotation about any axis is zero."

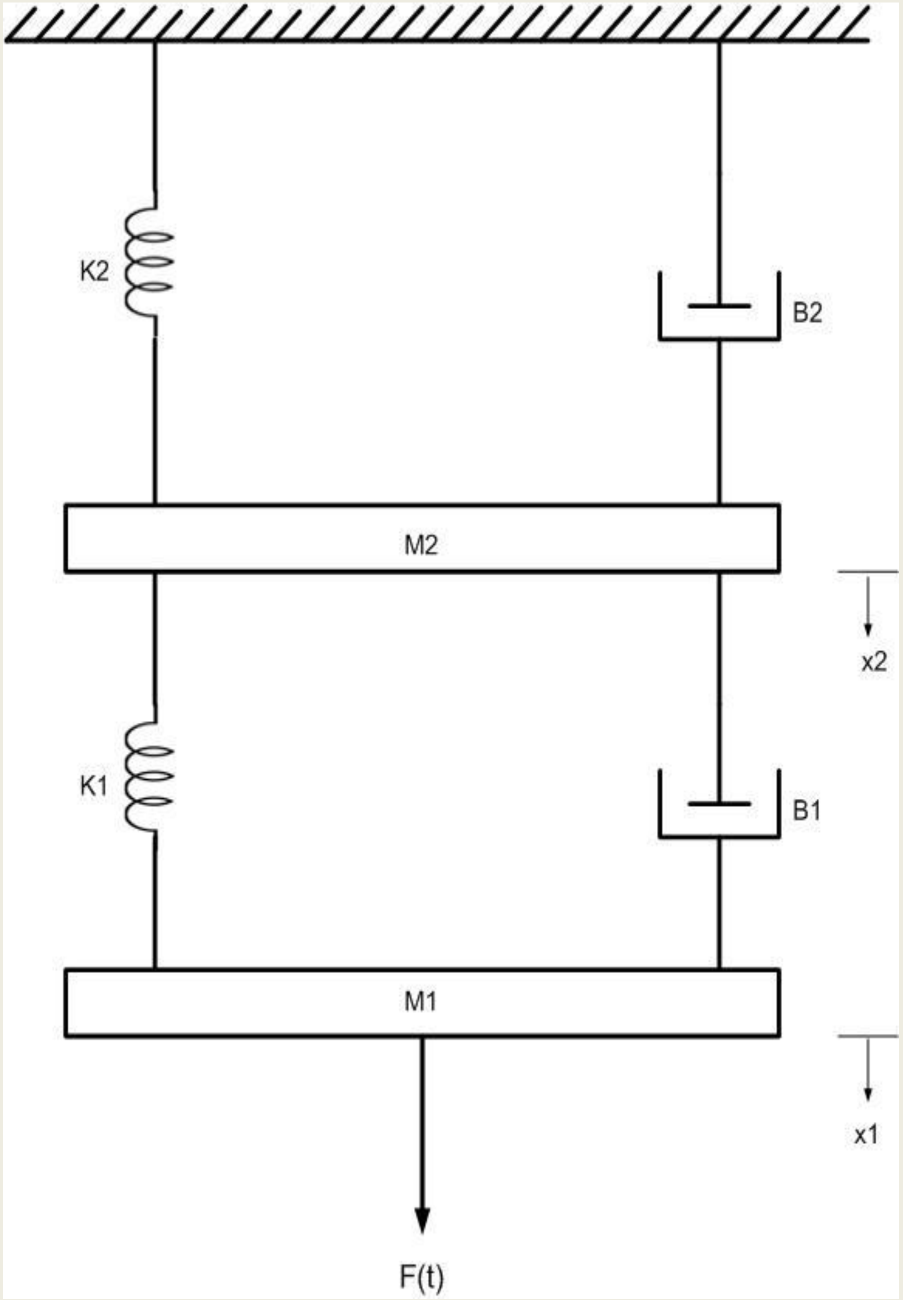
TANSLATIONAL-ROTATIONAL COUNTERPARTS

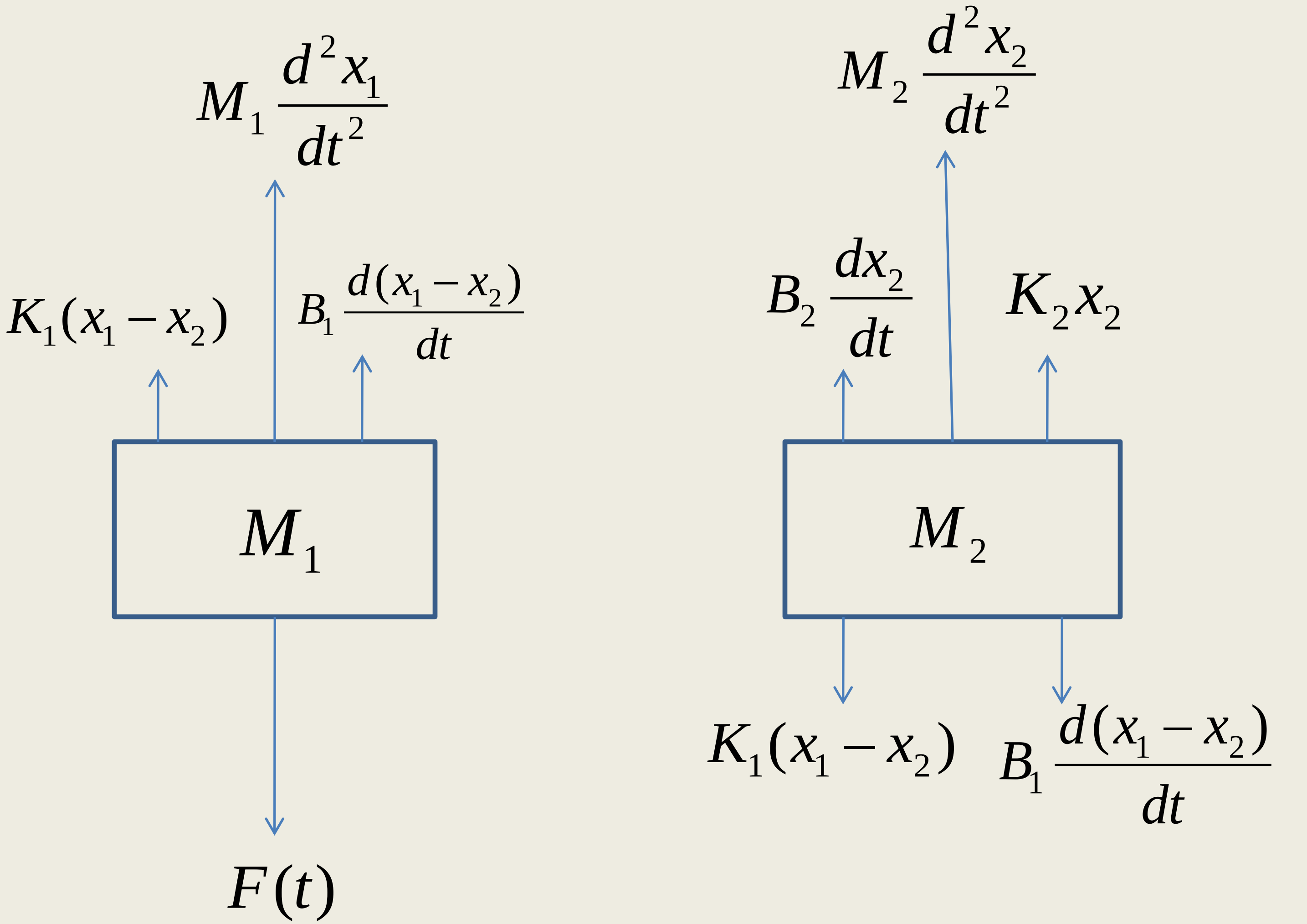
S.NO.	TRANSLATIONAL	ROTATIONAL
1.	Force, F	Torque, T
2.	Acceleration, a	Angular acceleration, α
3.	Velocity, v	Angular velocity, ω
4.	Displacement, x	Angular displacement, θ
5.	Mass, M	Moment of inertia, J
6.	Damping coefficient, B	Rotational damping coefficient, B
7.	Stiffness	Torsional stiffness

EXAMPLE: Draw the free body diagram and write the differential equation of the given system shown in fig.

Solution:

Free body diagrams are shown in next slide




$M_1 \frac{d^2x_1}{dt^2}$
$K_1(x_1 - x_2)$
$B_1 \frac{d(x_1 - x_2)}{dt}$
M_1
$F(t)$
$M_2 \frac{d^2x_2}{dt^2}$
$B_2 \frac{dx_2}{dt}$
$K_2 x_2$
M_2
$K_1(x_1 - x_2)$
$B_1 \frac{d(x_1 - x_2)}{dt}$

$$F_M = -M_1 \frac{d^2 x_1}{dt^2}$$

$$F_D = -B_1 \frac{d(x_1 - x_2)}{dt}$$

$$F_K = -K_1(x_1 - x_2)$$

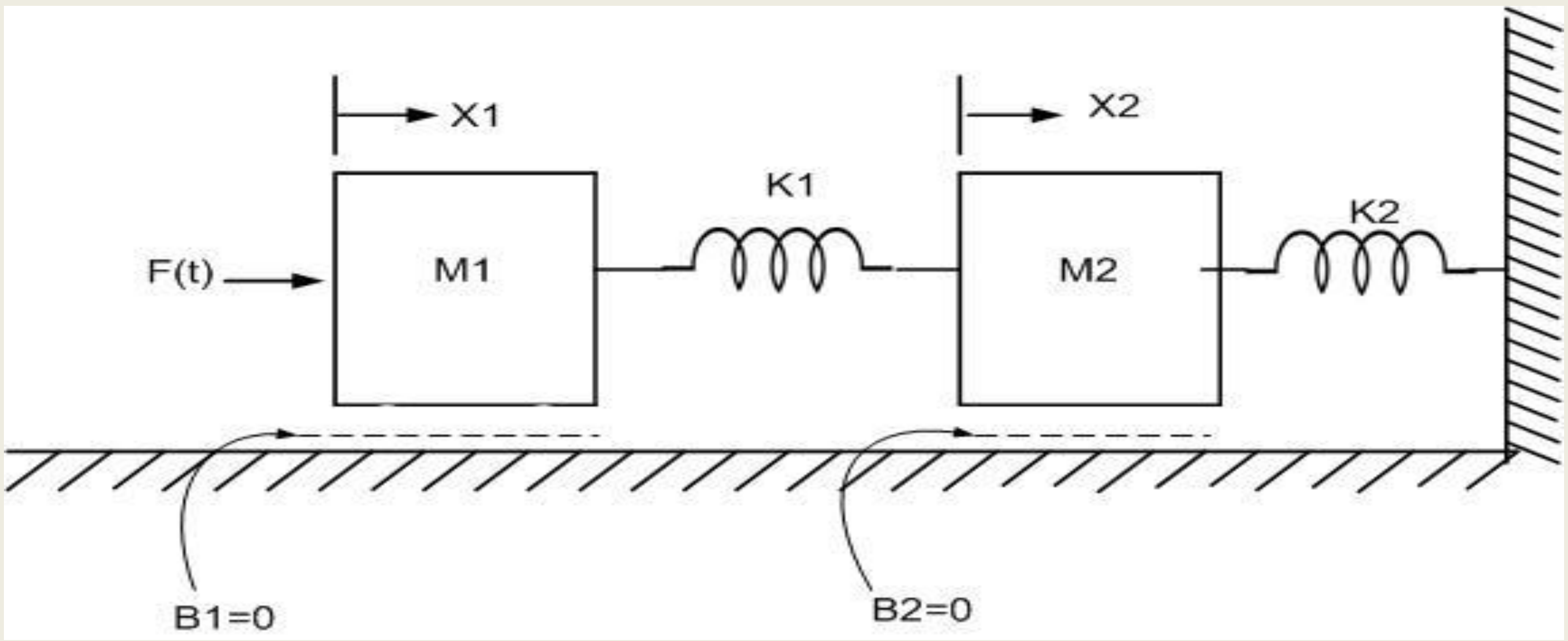
$$F(t) = M_1 \frac{d^2 x_1}{dt^2} + B_1 \frac{d(x_1 - x_2)}{dt} + K_1(x_1 - x_2)$$

similarly

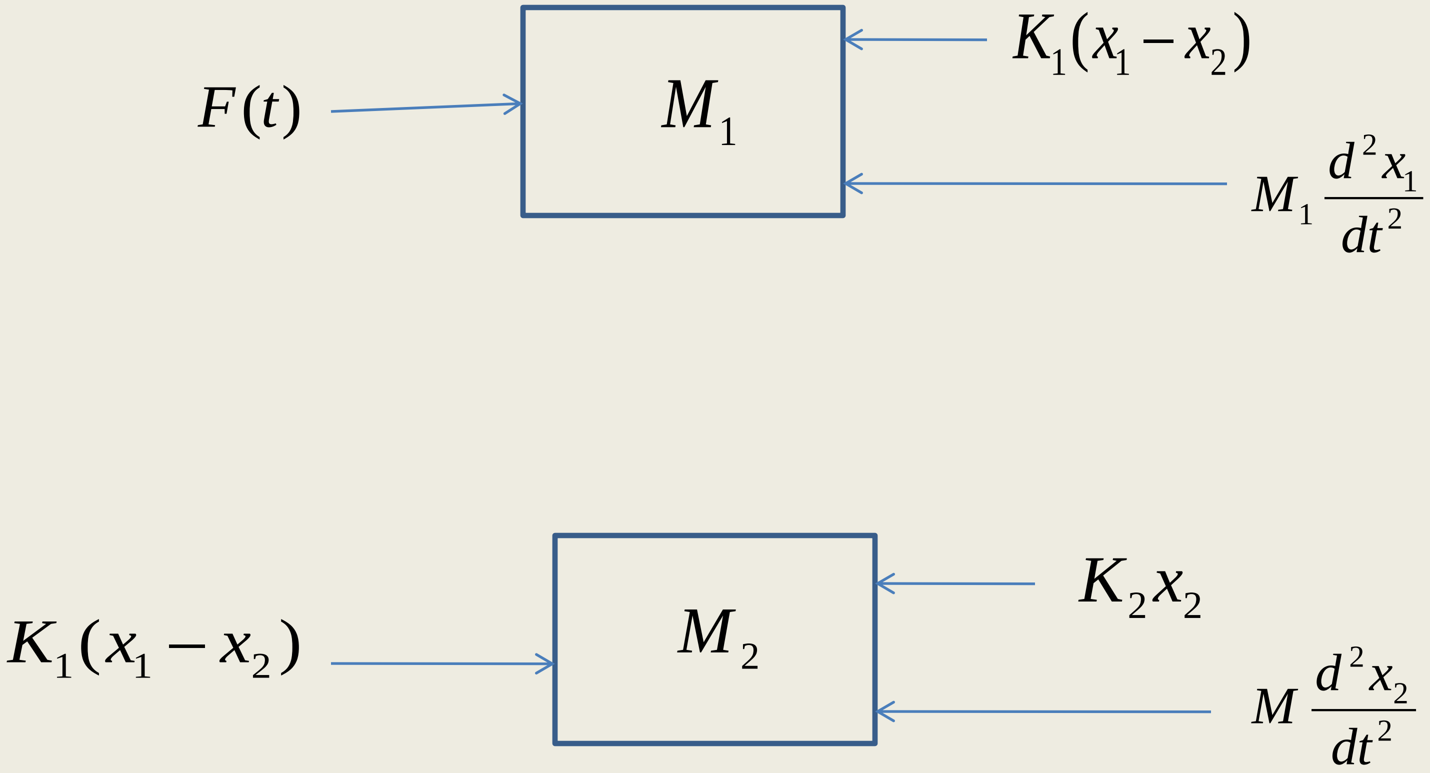
$$K_1(x_1 - x_2) + B_1 \frac{d(x_1 - x_2)}{dt} = M_2 \frac{d^2 x_2}{dt^2} + B_2 \frac{dx_2}{dt} + K_2 x_2$$

Write the differential equations describing the dynamics of the systems shown in figure and find the ratio $\frac{X_2(s)}{F(s)}$

Figure



FREE BODY DAIGRAMS



Differential equations are

$$F(t) = M_1 \frac{d^2 x_1}{dt^2} + K_1(x_1 - x_2)$$

$$K_1(x_1 - x_2) = K_2 x_2 + M_2 \frac{d^2 x_2}{dt^2}$$

Laplace Transform of above equations

$$F(s) = M_1 s^2 X_1(s) + K_1 X_1(s) - K_1 X_2(s)$$

$$K_1 X_1(s) - K_1 X_2(s) = K_2 X_2(s) + M_2 s^2 X_2(s)$$

Solve above equations for $\frac{X_2(s)}{F(s)}$

$$\frac{X_2(s)}{F(s)} = \frac{K_1}{(s^2 M_2 + K_1 + K_2)(K_1 + s^2 M_1) - K_1^2}$$

THANK YOU